This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

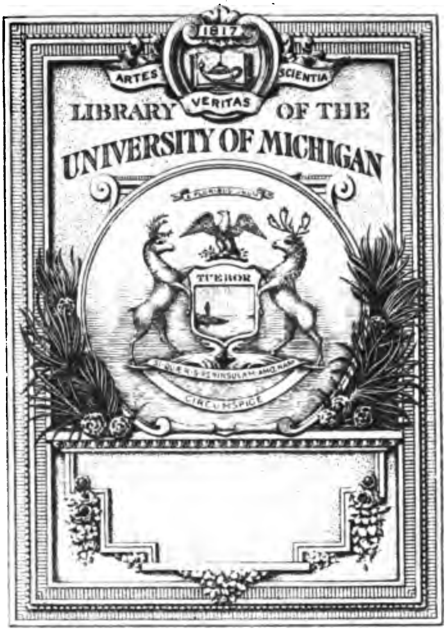
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`



LIBRARY OF THE
UNIVERSITY OF MICHIGAN

QC
19
.N63
175

AN

INTRODUCTION

TO

NATURAL PHILOSOPHY.

VOL. I.

AN

INTRODUCTION
TO
NATURAL PHILOSOPHY.

ILLUSTRATED WITH COPPER PLATES;

By WILLIAM NICHOLSON.

> Non enim me cuiquam mancipavi, nullius nomen fero: multum magnorum virorum judicio credo, aliquid et meo vindico. Nam illi quoque, non inventa, ſed quaerenda, nobis reliquerunt. SENECA.

THE SECOND EDITION, WITH IMPROVEMENTS.

IN TWO VOLUMES.

VOL. I.

LONDON:
PRINTED FOR J. JOHNSON, NO. 72. ST. PAUL'S CHURCH YARD.
MDCCLXXXVII.

SION COLLEGE
LIBRARY.

SOLD BY ORDER OF THE
PRESIDENT AND GOVERNORS 1938.

t. comm
dgon
-5.39
8627.
v.

TO

SIR JOSEPH BANKS, BART.
P. R. S.

SIR,

GRATITUDE and reſpect are due from every individual in ſociety, to him who promotes its real intereſt, by extending the bounds of knowledge. All Europe is acquainted with the exertions you have made for its advancement: permit me to join the voice of nations, by expreſſing the ſenſe I entertain of them. With this intention I take the liberty of dedicating the following treatiſe to the preſident of that reſpectable body of men, among whom the true philoſophy had its origin, and to whom it owes a great part of its improvements.

I am, with much reſpect,

SIR,

Your moſt obedient

humble ſervant,

LONDON,
Feb. 28, 1782.

W. NICHOLSON.

P R E F A C E.

THE advantages derived from the ſcience of Natural Philoſophy are ſo great, and ſo univerſally acknowledged, that an enumeration of them would be unneceſſary, if it did not ſerve to enliven and direct that ſpirit of enquiry which is natural to youthful minds, and to awaken thoſe, who from a want of reflection, are not inclined to look into the cauſes of things. We are apt to regard objects to which we have long been familiarized, with languor and indifference; and we now behold effects, without even the emotion of curioſity, which, in leſs enlightened ages, would have been thought miraculous.

Man, in a rude and ſavage ſtate, with a precarious ſubſiſtence, expoſed to the inclemencies of the ſeaſons, and the fury of wild beaſts, is an object of pity, when compared to man enlightened and aſſiſted by philoſophy. Ignorant of architecture, of agriculture, of commerce, and of all the numerous arts which depend upon the mechanic powers, he exiſts in the deſart, comfortleſs and unſocial, little ſuperior in enjoyment to the lion or the tyger, but much their inferior in ſtrength and ſafety. If it be true, that man ever exiſted in this ſtate, it could not have laſted long: the exertion of his mental ſtrength muſt have given riſe to the arts. Aided by theſe, the wilderneſs becomes a garden, embelliſhed with temples, palaces, and populous cities; and he beholds himſelf removed to an immenſe diſtance from the animals, to which in his original ignorance he ſeemed nearly allied.

The ſciences beſtow that leiſure and independence which have enabled ſuperior minds to form laws, and to eſtabliſh the rights of mankind, by mutual compact between the powerful and the weak. By this leiſure it is, that ingenious and ſpeculative men have collected maſſes of knowledge, which induce us to regard the powers of the human mind with aſtoniſhment. Hence we poſſeſs the admirable ſcience of Aſtronomy. A ſcience founded on the moſt accurate and long-continued obſervations, and ſyſtematized by the pureſt mathematical reaſoning; but at the ſame time ſo remote from vulgar apprehenſion, that its daily and important uſes and predictions are hardly ſufficient to prevent its being regarded by the ignorant as a chimera!

The other departments of Natural Philoſophy are not leſs replete with wonders. How great would have been the ſurpriſe of the antients, could they have foreknown the effects which are produced by the reflection and refraction of light? By a ſkilful management of theſe properties, teleſcopes, and various optical inſtruments are conſtructed. Objects, too remote to be perceived by the naked eye, are enlarged and rendered viſible. The ſatellites of Jupiter and Saturn, the mountains and cavities in the Moon, and the changes which take place on the Sun's diſc, are thus diſcovered, and afford matter for admiration and enquiry. Neither is this delightful ſcience of Optics confined to the contemplation of diſtant objects. Minute animals, the veſſels of plants, and, in ſhort, a new world in miniature is diſcloſed to our view by the microſcope, and an inexhauſtible fund of rational entertainment and knowledge is brought within the ſphere of our ſenſes.

Every one is acquainted with the benefits derived from the ſcience of Hydroſtatics, to which we are indebted for many uſeful inventions. Among theſe are wind and water-mills, pumps, fire-engines, ſteam-engines, &c. &c.

Chemiſtry is productive of great and ſingular advantages to ſociety. Metallurgy, in its utmoſt extent, the arts of making glaſs and pottery, of dying, and many others, together with a very conſiderable part of the materia medica, are dependant on this branch of philoſophy. The vaſt importance of metallurgy may be rendered obvious from the ſingle conſideration of the many uſes to which iron is applied. Without this metal we ſhould be almoſt totally incapable of making any utenſil or inſtrument. It is difficult to recollect any production of art in the formation of which iron is not made uſe of: and the very exiſtence of naval commerce depends on its magnetical property.

Philoſophy is not therefore a dry ſtudy, but a purſuit of the higheſt utility and entertainment, Thoſe who cultivate the ſciences know that they naturally produce a ſincere and diſintereſted love of truth. An enlarged view of things deſtroys the effects of prejudice, inſpires the propereſt ideas of the great original cauſe, and promotes a deteſtation of every thing that is mean or baſe. And if there be a pleaſure in attending to objects which fill the mind by their immenſity, and delight the imagination by the continual diſcovery of new and ſublime analogies, it is not to be wondered that philoſophers purſue their ſtudies with a degree of attention and ardor which is not found in any other ſet of men.

The order of arrangement in the preſent work is ſuch as was ſuggeſted by the ſubjects themſelves. After a curſory enumeration of the general properties of matter, Motion is principally attended to, being that affection of matter by which all changes are brought about. Mechanics and aſtronomy naturally follow; and are ſucceeded by an elucidation of the properties and motion of Light. The more complex motions of Fluids and the atmoſpheric phenomena are next conſidered. Thus far it will be obſerved, that the work treats of ſuch general effects as ariſe from the motions of bodies, without any particular reſpect to thoſe ſpecific properties which diſtinguiſh them into various claſſes. The remaining part of the treatiſe is employed upon theſe ſpecific properties: a long ſection upon Chemiſtry is given for the purpoſe of explaining them as far as they are at preſent known, and are capable of being underſtood by mere reading. Magnetiſm and electricity occupy the concluding ſections. Upon the whole, therefore, it will be ſeen, that the moſt ſcientific and beſt eſtabliſhed parts of Natural Philoſophy are firſt treated of, and are followed in ſucceſſion by others, which are leſs underſtood.

This treatiſe being intended to give a clear account of the preſent ſtate of Natural Philoſophy, to ſuch as poſſeſs very little mathematical knowledge, care has been taken to ſelect ſuch facts and experiments as tend to eſtabliſh elementary truths. The varieties of experiments of the ſame kind are not therefore numerous; but it is hoped that the advantage of a greater number of general principles is by that means obtained. Philoſophical inſtruments likewiſe are not minutely deſcribed. References to the parts of drawings are not often read or underſtood: for this reaſon, it was thought better to explain their general conſtruction, and leave the minutiæ to ocular inſpection. The grand object, throughout, has been to relieve the memory, and aſſiſt the underſtanding, by conciſeneſs and illuſtrative arrangement.

Thoſe prolix diſquiſitions, which render the commentator leſs intelligible than the author commented upon, are thus avoided: neither has the affectation of familiarity, which is uſually attended with a lax and unphiloſophical explanation of one event by another equally obſcure, been indulged. On the contrary, the author has every where endeavored to preſerve that ſolidity of argument, and preciſion of expreſſion, which diſtinguiſh the works of the beſt philoſophers. And, notwithſtanding the nature of the undertaking unavoidably required a deviation from thoſe elegant and general principles, which are obtained by ſtrict mathematical reaſoning, yet, it is preſumed, that the ſtudent will find nothing in this treatiſe which he will be under the neceſſity of unlearning, when he attempts the peruſal of thoſe books to which this is offered as an introduction.

The attentive examination of other books, to which the writer of this performance has had recourſe, has ſhewn him, that even the works of thoſe great men, who deſerve and poſſeſs the higheſt reputation, are not free from errors of importance. The preſent occaſion does not require the diſagreeable taſk of pointing them out; but this very conſideration will not permit him to hope that his diligence has entirely excluded miſtakes. However, he has little to fear on that account; being ſenſible that thoſe who are the beſt able to diſcover them, will, at the ſame time, be the readieſt to exerciſe that candor which every writer has need of.

The liberty which has been taken in altering the words of other authors, and adapting them to the purpoſe of this work, would have prevented the uſe of formal quotations, if they had been ſuppoſed neceſſary; and, as the preſent intention is not at all hiſtorical, the names of authors have been avoided as much as was conſiſtent with the wiſh of the writer to evade the ſuſpicion of plagiariſm. If plagiariſm can be imputed to the author of an epitome of ſcience, this acknowledgement muſt be allowed to obviate the charge.

In the printing, every thing which could be imagined of ſervice to the book, as a manual of philoſophy, has been done. A varying title at the head of each page, and copious indexes, are annexed. From theſe the reader will ſee, that ſcarcely any fact of importance has been omitted.

The learner, who may be induced to fix his chemical reading in his memory, by recurring to experiment, which may be done with very little expence, is cautioned to beware of the danger with which it is ſometimes attended. The ſolution, evaporation, and calcination of uninflammable matters may be performed in the common apartments of a dwelling-houſe; but the diſtillation of corroſive or inflammable ſubſtances ought not to be attempted but in a place prepared for the purpoſe. The burſting of a retort, containing any concentrated fuming acid, muſt be very deſtructive to furniture, as well as prejudicial to health; and ardent ſpirits, reſins, and the like, would endanger the houſe if a ſimilar accident were to happen. It is impoſſible to give advice againſt the many caſualties to which chemical experiments are liable; one general maxim is, always to endeavor, from analogy, to foreſee the conſequence, or probable reſult, of the intended proceſs, and when that cannot be done, to obſerve the phenomena, and proceed with caution.

ADVERTISEMENT.

THE Improvements in this ſecond Edition are very conſiderable; as well in conſequence of the rapid progreſs of the diſcoveries made by philoſophers in all parts of the civilized world, as of the careful reviſion the whole work has undergone. The additions are equivalent to a third volume; though, by an alteration of the type and page, this edition has been prevented from exceeding the former in bulk or price. Many additional figures have been inſerted in the plates, and the numbers 7 and 8 are entirely new. Marginal references are likewiſe annexed, which, it is hoped, will be found eminently uſeful. In theſe the figures denote the pages, and the letters the paragraphs of the pages where the proofs or illuſtrations referred to are to be found: and in the ſecond volume the numeral letter I. is prefixed, whenever the firſt volume is referred to.

London, Nov. 1, 1786.

ERRATA.

VOL. I. Page 2. line 14. read *part*. P. 84. l. penult. for BQ read RQ. P. 207. l. 5. r. *ſquares* of the diſtances. P. 217. l. 16. r. *nebula*. P. 277. l. 5. for 64 r. 82.

VOL. II. Page 15. line 18, for *ſiſh* read *ſink*. P. 17. l. 33. dele *the ſpecific gravity of*. P. 95. l. 28. for *or* r. *on*. P. 116. for *their* r. *other*. P. 119. The theorem was firſt diſcovered by Dr. Irvin of Glaſgow. P. 122. l. 18. for *melting or evaporation* r. *freezing or condenſation*. P. 143. l. 18. for *marine* r. *vitriolic*. P. 159. l. 10. dele *and the acidum perlati*.

CONTENTS

OF THE

FIRST VOLUME.

INTRODUCTION.

OF the degrees or kinds of knowledge; and the rules of philoſophizing, 1

BOOK I.

SECT. I.

Of Matter in the abſtract.

CHAP. I.

Of matter and its properties, 7

CHAP. II.

Of extenſion and impenetrability, 9

CHAP. III.

Of the inertia, and motion or reſt, 18

CHAP. IV.

Of attraction, conſidered chiefly as a power that generates motion, 25

CHAP. V.

Of the production, communication, or deſtruction of motion, 33

CHAP.

CHAP. VI.

Of the attraction of cohesion, and of ſpecific attractions, 44

BOOK I.

SECT. II.

Of bodies in motion.

CHAP. I.

Of the mechanical powers, 53

CHAP. II.

Of the lever, 55

CHAP. III.

Of the axis and wheel, and of the pulley or tackle, 60

CHAP. IV.

Of the inclined plane, and of the wedge, 64

CHAP. V.

Of the ſcrew, and of mechanical engines in general, 68

CHAP. VI.

Of the center of gravity, 74

CHAP. VII.

Of pendulums, 80

CHAP. VIII.

Of the motion of a body, which is acted upon by a centripetal force, 94

BOOK I.

SECT. III.

Aſtronomy.

CHAP. I.

Concerning the ſyſtem of the univerſe, 100

CHAP. II.

Of the figure and motion of the earth, 108

CHAP. III.

Of the mutual appearances of the ſuperior and inferior planets, 113

CHAP. IV.

Of the ſuperior planets, and of the true form of the planetary orbits, 118

CHAP. V.

Of the affections of the planets, 124

CHAP. VI.

Of parallaxes, and of the tranſit of Venus, 128

CHAP. VII.

Of the ſecondary planets, 137

CHAP. VIII.

Of the moon, 141

CHAP. IX.

Concerning the eclipſes of the ſun and moon, 145

CHAP. X.

Of comets; and of the proportion of light and heat on the planets, 159

CHAP. XI.

Of the teleſcopic appearance of the moon, 168

CHAP. XII.

Of the teleſcopic appearances of the ſun and the planets, 174

CHAP.

CHAP. XIII.

Of the length of days and nights; and of the ſeaſons, 180

CHAP. XIV.

Of the fixed ſtars, 189

BOOK I.

SECT. IV.

Concerning the phyſical cauſes of the celeſtial motions.

CHAP. I.

Of the general effects of centripetal forces on bodies in motion, 197

CHAP. II.

The univerſality of gravitation deduced from its effects, 207

CHAP. III.

Of the irregularities ariſing from the mutual gravitation of the planets, 218

CHAP. IV.

Of the figures of the planets; the preceſſion of the equinoxes, and the nutation of the earth's axis, 234

CHAP. V.

Of the tides, 241

BOOK II.

SECT. I.

Of Light.

CHAP. I.

Concerning the motion of light, 253

CHAP.

CHAP. II.

Optical definitions and principles, 261

CHAP. III.

Of the various refrangibility of the rays of light, 264

CHAP. IV.

Of the various reflexibility of the rays of light, 270

CHAP. V.

Concerning the rainbow, 273

CHAP. VI.

Of the ſeparation of the original rays of light by reflection or tranſmiſſion, that depends on the thickneſs of the medium upon which they are incident, 280

CHAP. VII.

General inferences reſpecting the diſpoſition to be reflected or tranſmitted, into which the rays of light are put, by the action that depends on the thickneſs of the medium upon which they are incident, 287

CHAP. VIII.

Of the permanent colours of natural bodies, and the analogy between them and the colours of thin tranſparent plates, 292

CHAP. IX.

Of the inflections of the rays of light which paſs in the vicinities of bodies, 298

CHAP. X.

Of the powers by which bodies reflect or refract the rays of light, 304

BOOK

BOOK II.

SECT. II.

Of Optics.

CHAP I.

Concerning the reflection and refraction of light by ſurfaces regularly formed, 315

CHAP. II.

Of dioptrics; or the regular refraction of light, 322

CHAP. III.

Of the eye; and of viſion, 327

CHAP. IV.

Of refracting microſcopes; or the dioptric inſtruments, by means of which ſmall and near objects are ſeen magnified, 335

CHAP. V.

Of refracting teleſcopes; or the dioptric inſtruments, by means of which remote objects are rendered large and diſtinct to the view, 342

CHAP. VI.

Of the imperfections of teleſcopes, and their remedies; and of the achromatic teleſcope, 348

CHAP. VII.

Of catoptrics, or the regular reflection of light; and of the reflecting teleſcope, 354

AN INTRODUCTION TO NATURAL PHILOSOPHY.

OF THE DEGREES OR KINDS OF KNOWLEDGE; AND THE RULES OF PHILOSOPHIZING.

THE impreſſions made on the organs of ſenſe by external objects produce ideas in the mind. We are continually employed in amaſſing a ſtock of general truths reſpecting them, which is called knowledge.

An intelligent being, whoſe powers are leſs than infinite, muſt of neceſſity be unequal to the performance of many things. If any aſſertion be made reſpecting two or more ideas, it will imply either truth or falſehood; but the caſes in which we can with certainty diſcover the one or the other are very few, in compariſon with thoſe that are placed beyond our power of inveſtigation. Yet, as a deciſion is almoſt always required for the regulation of our conduct in the affairs of life, the greater part of our knowledge becomes founded on probability, inſtead of eſtabliſhed truth. The means of acquiring knowledge may therefore be ſaid to be of three kinds. Certainties are obtained either by intuition or demonſtration; probabilities are obtained by analogy.

There are ſome ideas whoſe mutual relation in certain reſpects is ſo obvious, that nothing more is required to obtain the knowledge of it, than to apply them to each other. For example; if a given body be divided into parts, and the mutual relation between the whole body and one of its parts, with reſpect to magnitude, be demanded, the mind immediately conceives, with the cleareſt and moſt abſolute certainty, that the whole body is greater than its parts. If the particular body or magnitude in contemplation be abſtracted, or left out, the propoſition becomes general in this form, viz. every magnitude is greater than any part of the ſame. This kind of knowledge is called intuitive, and the general propoſitions are termed Axioms.

When it is required to determine the mutual relation of two ideas, whoſe agreement or diſagreement cannot be intuitively perceived, the truth may often be obtained by the interpoſition of a chain of axioms. This method of exhibiting the truth is termed demonſtration, and ſeems to be applicable only to ideas of the quantities and poſitions of magnitudes. For this reaſon, it will be difficult to give an example without having recourſe to the mathematics. The following will, however, be eaſily underſtood:

Fig. 1. Let the two circles in the figure be ſuppoſed equal, and the circumference of each to

paſs through the center of the other. Imagine the centers to be joined by the right line A B, and the lines C A, C B, to be drawn from one of the points where the circumferences interſect each other, to the centers reſpectively. Then, I ſay, the lines A B, B C, C A, will be equal each to each.

The demonſtration is as follows:

The word circle ſignifies a plane figure, contained under one line, called the circumference, to which all right lines drawn to a certain point within the figure, called the center, are equal. As ſoon therefore as it is underſtood that the figure A C D is a circle, and that the lines A B, C B, are right lines drawn from its center to its circumference, it is acknowledged intuitively, and without further argument, that thoſe lines are equal.

The ſame reaſon in the circle E C B evinces, that the lines A B, A C, are equal.

The lines A C, C B, being thus proved to be each equal to the line A B, are likewiſe equal to each other. For it is an intuitive truth or axiom, that things equal to one and the ſame thing are equal to each other.

The want of axioms, and the labour of demonſtration, are not the only impediments to the acquiſition of knowledge. Since knowledge is converſant with ideas only, it can be ſaid to poſſeſs reality with reſpect to external objects, ſo far only as thoſe ideas may be taken or ſubſtituted for the things they repreſent; and it is impoſſible to determine how far this may be done with propriety, even if it can be done at all. In referring from ideas to things we are liable to error, not only becauſe the compound idea of a being conſiſts of an aſſemblage of its properties, which may be incomplete and inadequate, but likewiſe becauſe thoſe ideas may even be quite different from any thing exiſting in the being itſelf, as may be inſtanced in the ideas of colour, ſound, pain, &c. The great perſpicuity and certainty of mathematical knowledge ariſes from the ſimplicity of the ideas employed, and their not depending on any external being: for, as this ſcience treats only of ideas, it is of no conſequence to its truths, whether geometrical figures ever had an exiſtence; it being ſufficient that their exiſtence is poſſible.

The greater number of our ideas being too complex and imperfect to admit of intuitive concluſions or axioms, it is evident, that in general we muſt be contented with leſs proof than demonſtration. Inſtead therefore of endeavouring to obtain axioms by comparing ideas, we obſerve events, and from the contemplation of what has happened, we form a preſumption of what will again come to paſs. Obſervation has ſhewn us, that a certain event is always followed by another determinate event; we ſuppoſe a relation to ſubſiſt between them; we imagine this relation to be neceſſary; we diſtinguiſh the prior event by the name of Cauſe, and the latter we call the Effect. This kind of knowledge, which is not founded on reaſoning, but on experience alone, may be termed Analogical, and is much leſs perfect than what is obtained by intuition or demonſtration. That a ſtone will deſcend to the earth, is an analogical propoſition. It cannot be demonſtrated: but, from the conſideration of a vaſt number of events of the ſame nature, a degree of probability ariſes, which commands our aſſent. It is clear, that analogical propoſitions are no more than ſtrong probabilities, from the remarkable circumſtance, that their converſe does not imply an abſurdity. To deny an intuitive or demonſtrative truth, is to aſſert an impoſſibility; but to deny an analogical truth, is only to aſſert an improbability. The underſtanding revolts at the affirmation, that a part is greater than the whole body; but we ſee no impoſſibility in the aſſertion, that a ſtone, at ſome time or place, has remained in the air without a tendency to deſcend; this ſuppoſition being highly improbable, but nothing more. In fact, demonſtration is a collection of truths or axioms; analogy is a collection of probabilities. Simple probabilities are to analogy what axioms are to demonſtration. Now, there is no compariſon in point of certainty between axioms; all being equally true; but probabilities differ exceedingly in their degree of credibility.

Natural Philoſophy, ſtrictly ſpeaking, admits of no other proofs than thoſe of analogy. To give ſtability to this ſcience, it is neceſſary to admit no probabilities as firſt principles of analogy, but thoſe which poſſeſs the ſtrongeſt and moſt incontrovertible reſemblance to truth. For this purpoſe, the following rules are adopted:

Rules of Philoſophizing.

I.

No more cauſes of natural things ought to be admitted than are true, and ſufficient to explain the phenomena.

II.

And therefore effects of the ſame kind are produced by the ſame cauſes.

III.

Thoſe qualities, whoſe virtue can neither be increaſed nor diminiſhed, and which are found in all bodies with which experiments can be made, ought to be admitted as qualities of all bodies in general.

BOOK I.

SECT. I.

Of Matter in the abſtract.

CHAP. I.

OF MATTER AND ITS PROPERTIES.

A MATTER is known to us only by its properties.

B The common properties of matter are extenſion, impenetrability, inertia, attraction, motion, and reſt; all which, except the two laſt, which cannot exiſt together, are found in all bodies whatſoever.

It would be, perhaps, a fruitleſs attempt, to
enquire whether theſe are the only qualities with
which bodies are endued in common. Matter may
poſſeſs many others, that our ſenſes are not adapted
to obſerve, or which have hitherto eſcaped the
notice of Philoſophers. But it is neceſſary to C
obſerve, that we are totally ignorant of the ſub-
ſtratum in which theſe properties are united. The
eſſence of matter is unknown to us. We muſt
not, therefore, aſſume one or more of theſe pro-

perties as compoſing that eſſence itſelf; for errors of the greateſt importance have ariſen from this ſource.

D There are other properties, ſometimes called ſpecific, that are not found in all bodies; as tranſparency, opacity, fluidity, conſiſtence, and the like. But theſe ſeem to relate to the figures or motions of the parts of bodies, and are, therefore, referable to the general properties. There are alſo ſeveral ſpecies of attraction and repulſion, which will be attended to in their proper places.

Here follow definitions of the general properties abovementioned.

E Extenſion is that affection of matter by which it occupies part of ſpace.

F Impenetrability, is that by which two bodies cannot exiſt in the ſame place at the ſame time.

G Inertia, is that by which a body reſiſts any force impelling it to a change of ſtate; i. e. of motion or reſt.

H Attraction, is that by which one body continually tends to approach to, and, if not by external means prevented, does approach to, another body or bodies.

I Motion, is a continual and ſucceſſive change of place. Reſt is the permanency or remaining of a body in the ſame place.

CHAP. II.

OF EXTENSION AND IMPENETRABILITY.

THE idea of extenſion is ſo ſimple, that it K
cannot be defined. For though in the preceding
Chapter it was pointed out as that affection
by which matter occupies part of ſpace, yet, there
can be little doubt but that the idea of extenſion is
itſelf antecedent to that of ſpace, and therefore not
definable by it. In order to facilitate the conſideration
of thoſe truths that relate to extended magnitudes,
geometers have, as it were, analyzed extenſion.
It is evident that extenſion implies form or L
figure, and figure muſt be limited. The limit or
termination of figure is called a ſurface, or ſuperficies.
A ſuperficies is likewiſe limited, and its
termination is called a line. And the termination
of a line is called a point. Now, though it is clear, M
that a ſuperficies, a line, or a point, cannot exiſt
ſeparate or apart from an extended being, yet, it
is certain, that the ideas of them may be conſidered
diſtinctly, without immediately referring to the
other conſequences ariſing from the general idea of
extenſion. In this ſenſe mathematicians define a N
point to be that which has no part, or is altogether
indiviſible; a line to be that which is length,
without breadth, or is diviſible in one reſpect,
namely, of length; a ſurface to be that which has
only length and breath, or is diviſible in two reſpects,
namely, of length and breadth; and a ſolid

to be that which has length, breadth, and thickneſs, or is diviſible in three reſpects, namely, of length, breadth, and thickneſs.

o No finite or imaginable diviſion of a line can ever produce a point or indiviſible part. A line is therefore diviſible into an infinite number of other lines, or ſimilar parts, and conſequently much more is a ſuperficies, and yet more a ſolid. A mathematical ſolid, that is to ſay, pure extenſion, is diviſible to infinity: and if the elements of matter be of the ſame nature as the aggregates they compoſe, matter is likewiſe infinitely diviſible. This being ſuppoſed, the following theorems will be eaſily underſtood.

p THEOREM I.

Any quantity of matter, how ſmall ſoever, and any finite ſpace, how great ſoever, being given (as for example, a cube circumſcribed about the orb of Saturn) it is poſſible for the ſmall quantity of matter to be diffuſed throughout all that ſpace, and to fill it, ſo that there ſhall be no pore or interſtice in it, whoſe diameter ſhall exceed a given finite line.

Let the cube in queſtion be divided into ſmall cubes, whoſe ſides ſhall each be equal to half the given line. It is eaſily ſeen, that the number of ſmall cubes will not be infinite. Imagine the quantity of matter to be divided into a number of parts equal to that of the ſmall cubes, and place a particle in the center of each cube. The whole ſpace will thus be filled, and the greateſt diſtance between two adjacent particles, or, in other words, the diameter of any pore or interſtice, will be leſs than the given line.

COROLLARY.

Hence there may be given a body, whoſe matter, if it be reduced into a ſpace abſolutely full; that ſpace may be any given part of its former magnitude.

THEOREM II. Q

There may be two bodies of equal bulk, whoſe quantities of matter being unequal in any proportion; yet the ſum of their pores, or quantity of void ſpace in each of the two bodies, ſhall be nearly in the ratio of equality to each other.

This laſt theorem is not ſo obvious as the former, but an inſtance will render it eaſy.

Suppoſe one thouſand cubic inches of gold to contain one cubic inch of matter, or, in other words, when reduced into a ſpace abſolutely full, to be equal to one cubic inch; then one thouſand cubic inches of * water will contain one nineteenth part of an inch of matter when reduced. Conſequently, the void ſpaces in the gold will be nine hundred and ninety-nine cubic inches, and thoſe in the water nine hundred and ninety-nine cubic inches, and eighteen nineteenth parts of an inch; that is, they will be nearly in the ratio of equality.

Yet, the actual diviſibility of matter can pro- R bably be carried but to a certain degree. The ultimate particles of bodies, it is moſt likely, are not to be altered by any force in nature. But there

---

* Gold is nineteen times as heavy as water.

are, nevertheleſs, many inſtances which ſhew to what inconceivably minute parts bodies may be actually divided.

S A grain of leaf-gold will cover fifty ſquare inches, and contains two millions of viſible parts; but the gold which covers the ſilver wire, uſed in making gold lace, is ſpread over a ſurface twelve times as great.

T In making this wire, it is uſual to gild a cylindrical bar of ſilver ſtrongly, and afterwards draw it into wire by paſſing it ſucceſſively through holes of various magnitudes in plates of ſteel. By this means the ſurface is prodigiouſly augmented; notwithſtanding which, it ſtill remains gilt, ſo as to preſerve an uniform appearance even when examined with the microſcope. The quantities of gold and ſilver, and the dimenſions of the wire, are known. With theſe data it is eaſy to calculate, and from calculation it is proved, that ſixteen ounces of gold, which, if in the form of a cube, would not meaſure one inch and a quarter in its ſide, will completely gild a quantity of ſilver-wire ſufficient to circumſcribe the whole globe of the earth.

U The animalculæ obſerved in the milt of a codfiſh are ſo ſmall, that many thouſands of them might ſtand on the point of a needle.

Suppoſing the globules of the blood in theſe animalculæ to be in the ſame proportion to their bulk as the globules of a man's blood bear to his body, it appears, that the ſmalleſt viſible grain of ſand would contain more of theſe globules than

10,256 of the largeſt mountains in the world would contain grains of ſand.

Theſe inſtances may ſerve to ſhew the amazing v
fineneſs of the parts of bodies, which are nevertheleſs ſtill compounded. Gold, when reduced to the thinneſt leaf, ſtill retains thoſe properties which ariſe from the modification of its parts. Microſcopic animalculæ are without doubt organized bodies, and the globules of their blood are poſſeſſed of ſpecific qualities. Even the rays of light are compounded of an almoſt infinite variety of particles, which, when ſeparated from each other, exhibit the powers of exciting ideas of colours. None of theſe are the ultimate particles of which all bodies are formed, for they all bear evident marks of compoſition. How inconceivably ſmall then muſt thoſe particles be!

To theſe ultimate particles alone it is, that im- w
penetrability can be attributed. Penetration takes place in all compounded bodies. Water exiſts in the pores of wood. Air in the pores of water. Quickſilver in the pores of gold, &c. &c.

Some philoſophers have queſtioned whether im- x
penetrability be really a property of matter; and it muſt be confeſſed, that, notwithſtanding this idea is ſo cloſely connected in the formation of our compound idea of matter, yet, if we examine from whence the notion is originally obtained, we ſhall find that our knowledge is much leſs certain than we may have ſuſpected.

To make this clearer, we muſt conſider that our y
notion of impenetrability is derived from the ſenſe

of feeling. We move the hand towards a body; and in ſituations where motion is not generated, it is prevented by that body from going forward; from which we conclude, that the body poſſeſſes a part of ſpace to the excluſion of every other body; that is to ſay, that it is impenetrable.

z But, in order to juſtify this concluſion, it is neceſſary that we ſhould be certain that it is the body itſelf, actually occupying ſpace, which reſiſts the preſſure; and of this we cannot be aſſured, ſince we obſerve many inſtances in which bodies afford reſiſtance to other bodies which move in ſpaces at ſome diſtance from the reſiſting body. Thus, the loadſtone, in certain circumſtances, reſiſts the motion of iron which approaches towards it; and there is no doubt but this reſiſtance or repulſion, if exerted on any part of a man, would afford a ſenſation ſimilar to that which ariſes from contact. If the man had not ſight, or ſome other ſenſe to perceive that the reſiſting body was really diſtant, he would, from the ſenſe of touch, conclude that the body was in contact with the part perceiving; and, if any force he could produce were inſufficient to overcome that reſiſtance, he would conclude the body to be impenetrable.

A Now, by ſeveral experiments, which we ſhall have occaſion to mention in the courſe of this work, there is the higheſt reaſon to conclude, that all bodies exert a repulſive force on each other, and that the common effects which are attributed to contact and colliſion are produced by this repulſion:

ſion: And, if ſo, why not attribute all effects of the ſame nature to this cauſe, which we know exiſts, inſtead of ſuppoſing an impenetrability that can never be proved?

B If the force of repulſion be ſufficiently great, it may not be in the power of any natural agent to overcome it, and, conſequently, all the effects of a real impenetrability will take place, though the ſubſtratum or matter itſelf may not be impenetrable, or even extended.

C It is not in our power to determine, whether impenetrability or extenſion be eſſentially neceſſary to exiſtence. For the extenſion of the elements of matter ſeems capable of no other proof than what may be drawn from their impenetrability; and experiment cannot decide, becauſe a finite preſſure can only prove the reaction of a finite reſiſtance.

D The queſtion therefore is, whether it be more probable, that the particles of matter are beings poſſeſſed of a finite power of repulſion, which prevents their mutual approach, but does not render mutual penetration or coincidence in the ſame part of ſpace impoſſible, on the application of force ſufficient to overcome that repulſion; or whether they be impenetrable atoms, which, conſequently, muſt reſiſt ſuch coincidence with an infinite force?

E Here we muſt attend to the facts. If the repulſion continually increaſed as the diſtance of the bodies decreaſed, we might conclude, that it was the only cauſe of the apparent impenetrability of bodies; but, as in the loadſtone, there is a certain ſmall diſtance at which repulſion ceaſes, and attraction takes place, ſo in compreſſing bodies together, with a certain degree of preſſure, the diſtance is at length diminiſhed ſufficiently for the bodies to adhere. The phenomena are probably ſimilar; but, at all events, the coheſion of the parts of bodies ſhews a mutual attraction; and it is not eaſy to explain why the parts ſhould not mutually penetrate and coincide, when the repulſion on which their impenetrability was ſuppoſed to depend has ceaſed, and given place to attraction. And on this account, the doctrine of impenetrable atoms ſeems the moſt probable.

This, however, ſuppoſes the impenetrable particles to come into contact, where attraction has taken place. But it is certain they do not. For things in abſolute contact cannot come nearer without penetration, and cold is known to diminiſh the bulk of bodies; or, in other words, to bring their parts nearer each other than before. This contraction is greater, the greater the cold. Without enquiring into the nature of cold, we may therefore preſume that the utmoſt poſſible cold would either bring the impenetrable particles into abſolute contact, or cauſe the body to vaniſh by the mutual penetration of all its elements. Thus, the original queſtion returns to us, and the only remaining argument ſeems to be——If by the firſt rule of philoſophizing (6) we are to admit no more cauſes of natural things than are ſufficient to explain the phenomena, and we know that a ſphere of repulſion exiſts as the proximate cauſe of our ideas of impenetrability and extenſion, why ſhould we add to this an extended atom exiſting in the center of the ſphere of repulſion?

The quantity of matter in the univerſe is much G
leſs than is generally imagined. This may be deduced from what has been ſaid already on this ſubject; but more eſpecially from the properties of tranſparent bodies. Light paſſes through theſe in all directions without the leaſt difficulty. The focus of a burning mirror, which augments the denſity of the ſun's rays upwards of three thouſand times, may be received in the bodies of glaſs or water, without producing any effect; ſo far are the particles of thoſe ſubſtances from impeding the paſſage of light. And the bottom of the ſea has been diſcovered at a greater depth than ſixty feet. It is not improbable that the real matter in a ſmall piece of glaſs may bear a leſs proportion to its bulk than that bulk does to the whole earth. Whence the electric matter paſſes with an unmeaſurable velocity through the pores of gold and other bodies, and the magnetic power exerts itſelf undiminiſhed through all ſubſtances, iron excepted.

To render the poſſibility of this more evident H
let a body be ſuppoſed to be ſo conſtructed, as to have as much vacuity as matter; then half the body would be vacuous. Suppoſe the particles of which it is compoſed to be conſtructed in the ſame man-

ner; then the vacuity becomes three-fourths of the ſpace occupied by the body. Again, let theſe laſt mentioned particles be conſtructed in the ſame manner; the vacuity will then be ſeven-eighths. And the ſeries being carried forward to the tenth order of particles, the vacuity will exceed the matter one thouſand and twenty-three times.

CHAP. III.

OF THE INERTIA, AND MOTION OR REST.

IT is chiefly from the inertia that we obtain a knowledge of the relative quantities of matter in bodies. The quantities of matter in bodies abſolutely ſimilar in compoſition, are determined by their extenſion; but in diſſimilar bodies the ratio does not hold. Now, in bodies ſimilar in compoſition, we obſerve that the inertia follows the proportion of the extenſion, that is, by reaſon of the ſimilarity; the proportion of the maſs of matter; and from thence, by applying the proportion of the inertia to diſſimilar bodies, we obtain a knowledge of their maſſes. Thus, for example, the quantity of matter in one cubic inch of gold is as 1, in two cubic inches as 2, in three cubic inches as 3, and ſo forth: this we gather from the extenſion, and alſo from the inertia, both which, in this caſe, follow the ſame proportion. But if a cubic inch of copper be added, though the extenſion be augmented as 1, yet the inertia increaſes only as $\frac{1}{2}$; therefore, either the extenſion or the inertia is not the proper meaſure of the maſs; and, as we can more readily conceive that the extenſion, or ſpace occupied within the external limits of the body, may, by poroſity in the body, ceaſe to be the meaſure, than that the inertia of the ultimate parts of matter ſhould vary; we conclude, that the quantity of matter is as the quantity of the inertia; though it muſt be allowed that neither poſition is phyſically demonſtrable.

The inertia of matter being that by which it K reſiſts any change of ſtate with regard to motion or reſt, is meaſured by the force which is required to produce a given change; that is to ſay, the force required to give a certain degree of velocity to a body at reſt A, containing ten parts of matter, is five times as great as would produce the ſame effect on a body at reſt B, containing two parts.

This force in a moving body is called the quan- L tity of motion, or momentum, and is meaſured by the maſs of matter multiplied by the velocity; for the whole motion of a body is the ſum of the motions of all its parts. Therefore, in the laſt mentioned inſtance, the body A moves with five times the force that B moves with, though the velocity is the ſame in both. But if the velocity of B were to be augmented five times, the quantities of motion would then be equal; that of A being expreſſed by 10, multiplied by 1, and that of B being expreſſed by 2, multiplied by 5.

Motion and reſt are diſtinguiſhed into abſolute M and relative. Abſolute motion is the removal of a N body from one part of ſpace to another. Relative motion is a ſucceſſive change of ſituation with reſpect to another body, though that body may not be at reſt. Thus, a man ſitting in a barge in motion, is relatively at reſt, that is, with reſpect to the parts of the barge: but abſolutely in motion; being removed, with the veſſel, from one part of ſpace to another. On the contrary, the bargeman, who fixes a ſtaff in the ground, and gives motion to the barge by walking along its gunwale, is abſolutely at reſt, for the ſtaff againſt which he leans is fixed; but relatively in motion, ſince, with reſpect to the veſſel, he walks from one end to the other. But if the earth be ſuppoſed in motion, the abſolute motion of the barge and its contents will be compounded of its relative motion, together with the abſolute motion of the earth.

O There is another diſtinction in motion, by which it is called apparent or angular, and which depends on an optical fallacy. Thus, to an eye at E, (fig. 2.) a body which moves from C to D, or from F to G, will apparently deſcribe the line A B, though the real motions are very different. And if a body move, either directly towards, or directly from the eye, it will be apparently at reſt. It is true, that, from other circumſtances, we have acquired the habit of diſtinguiſhing different motions which are made obliquely to the eye; but where thoſe circumſtances are wanting, as in the heavens, it requires no ſmall degree of attention to diſtinguiſh the real from the apparent motion.

Experience proves, that the three following laws are ſufficient to account for all the phenomena of motion.

LAW I.

Every body continues in its ſtate of reſt, or of uniform motion in a right line, unleſs compelled to change that ſtate by forces impreſſed.

For matter at reſt, being endued with no power of moving itſelf, would remain ſo for ever, unleſs impelled by ſome external cauſe.

We have alſo daily proofs, that a body in motion will continue to move uniformly in a right line, unleſs prevented or diverted by ſome other agent. The reſiſtance of the air, and the force of gravity, in a ſhort time deſtroy the motion of all projectiles, which otherwiſe, by the vis inertiæ, would continue for ever.

LAW II.

All change of motion is in proportion to the force impreſſed, and is made in the line of direction in which that force is impreſſed.

For if any force produces motion, a double force will produce a double quantity, and a triple force a triple quantity, whether it be impreſſed all at once, or by ſucceſſive impulſes. And this motion, ſince it always has the ſame direction as the generating force, if the body be already in motion, either increaſes the ſame, by conſpiring therewith, or diminiſhes it by oppoſition, or is added to it obliquely, being compounded with it according to the directions of the two motions.

LAW III.

Action and reaction are always equal and contrary; or, the mutual actions of two bodies are always equal, and in contrary directions.

Thus, if a ſtone be preſſed by the finger, the finger is equally preſſed by the ſtone. If a horſe draws a ſtone, the ſtone draws the horſe equally backwards, for the rope is equally ſtretched towards both. If one body impels another, it will itſelf ſuffer an equal change of motion by the reaction in a contrary direction. If the loadſtone attract iron, it is itſelf equally attracted, and both are at reſt, when they come together, which could not be if they did not preſs equally. By this means the changes of motion, though not of velocity, are always equal.

To illuſtrate this yet more. Suppoſe a horſe proceeds with a quantity of motion expreſſed by the number 3, and that it would require a force equal to 2 to move a certain ſtone. The horſe then drawing, proceeds with a force equal to 1, the reaction of the ſtone deſtroying as much force as the action communicates to it.

From theſe laws the following corollaries are eaſily deduced, which may be applied to ſolve all the effects which can be produced by the mechanical powers.

COROLLARY I.

A body impelled by two forces acting in the direction of the two ſides of a parallelogram will deſcribe the diagonal in the ſame time as, by the action of one of the forces, it would have deſcribed one of the ſides.

A body at A (fig. 3.) would be carried with an uniform motion in a given time to B, by the ſingle force M impreſſed at A; and by the ſingle force N, impreſſed at the ſame place, would be carried from A to C: complete the parallelogram ABDC, and by the combined forces, it will be carried in the ſame time in the diagonal from A to D. For ſince the force N acts according to the direction of the line AC, which is parallel to BD; it will, by Law II. in no reſpect alter the velocity of approaching to BD, which was produced by the other force. Therefore, the body will in the ſame time arrive at the line BD, whether the force N be impreſſed or not; and at the end of the time will be found ſomewhere in the ſaid line BD. By the ſame manner of arguing, it will at the end of the time be found ſomewhere in the line CD, which muſt of conſequence be in that place where they interſect each other. Its motion will be in a right line by Law I.

COROLLARY II.

Hence alſo appears the compoſition of a direct force AD, (fig. 3.) from any two oblique forces AB and BD, and on the contrary, the reſolution of a direct force into any two oblique forces, AB and BD.

The laws of motion being obvious deductions from the phenomena around us, are confirmed by every mechanical effect we ſee produced. In the conſideration of forces, it is very often convenient to regard them as if compounded of two or more forces, as will be ſhewn in the enſuing ſection of this Book. It is clear, that any given motion or force AD may be produced by any pair out of an indefinite number of pairs of forces that may be compounded together; for example, AC and AB, or AF and AE, or, generally, by any two forces, whoſe quantity and direction may conſtitute a parallelogram, having AD for its diagonal. In this parallelogram are ſix things, namely, the directions of the two compounding forces, and of the compounded force, and alſo their reſpective quantities. And if any four of theſe be known, the other two may eaſily be found, by completing the conſtruction of the figure.

COROLLARY III.

U The quantity of motion which is obtained by taking the ſum of the motions made in the ſame direction, or the difference of thoſe made in contrary directions, is not changed by the mutual action of the bodies.

For action and reaction being equal, by Law III, (22, R) will therefore occaſion equal changes in the motions, but in contrary directions. Conſequently, if the motions are both made in the ſame direction, whatever is added to the motion of the impelled body muſt be ſubtracted from that of the impelling body, and the ſum will remain the ſame. But if the bodies meet directly, the quantity of motion deſtroyed being equal in each, the difference of the motions made in contrary directions will remain unchanged.

CHAP. IV.

OF ATTRACTION, CONSIDERED CHIEFLY AS A POWER THAT GENERATES MOTION.

v THE force which tends to bring bodies together, without any hitherto diſcoverable impulſe, is called attraction. Whether the various kinds of attraction that fall under our obſervation may be referable to one common immediate cauſe, cannot yet be determined. w The third rule of philoſophizing (6) authorizes us to reckon it among the properties of matter; and whether we ſhall ever proceed ſo far as to diſcover, that it is ſecondary to ſome other more remote property, muſt depend on future reſearches. x It ſeems clear, however, that it is not deducible from any of the properties we have enumerated, (7, B) and conſequently that all ſuppoſitions reſpecting the circulation of effluvia, are mere words without meaning.

y The ſeveral kinds of attraction are, the attraction of gravitation, or gravity; the attraction of coheſion; the attraction of combination, or chemical affinity; the attraction of magnetiſm, and the attraction of electrized bodies. In this place we are to conſider only the attraction of gravitation as a power that generates or produces motion.

Z Gravitation is that force by which bodies fall to each other. The vicinity of the earth, which ſtrongly attracts every thing to itſelf, prevents its effects between ſmaller bodies from appearing; but the attraction of the mountain of Schehallien in Scotland, upon the ball of a pendulum was found by a very accurate ſet of obſervations to be conſiderable.

A This power is found to act on all bodies, at any given place, preciſely according to the quantity of matter in each, which is diſcovered by the vibration of pendulums, thus. Let the two unequal bodies A and E (fig. 4.) be ſuſpended by threads of the ſame length, and be let fall at the ſame time from the points A and E, in the arcs A C and E G, which are at equal diſtances from the two loweſt points D and H. Then the vibrations of each will be performed in equal times, and conſequently the velocities will be equal. Whence the quantity of motion in each, being the product of its maſs of matter multiplied by its velocity (19, L) will be in proportion to the maſs of matter in each. But (21, Q) the force producing motion is in proportion to the quantity of motion produced. Therefore, the force of attraction is in proportion to the quantities of matter in bodies.

B This likewiſe appears in falling bodies, all which being let go from equal heights, how different ſoever in weight, arrive at the ground in the ſame time, that is, with quantities of motion in proportion to their reſpective quantities of matter.

The reſiſtance of the air is not here conſidered, for the ſake of perſpicuity, though it very ſenſibly impedes all motions performed in it. A guinea will arrive at the ground in leſs time than a feather; but in the receiver of an air-pump, out of which the air is exhauſted, they both fall in the ſame time.

Gravitation acts on all bodies at all times, and C
that equally, whether in motion or at reſt, as is evident from the velocities of falling bodies, which are uniformly accelerated during the whole of their courſe. That a force conſtantly and equally acting, D
will produce an uniform acceleration of velocity, is plain, from the following conſiderations. Suppoſe a body A, begins to move, by the impulſe of gravity impreſſed at that inſtant, with a velocity expreſſed by the number 1, the next inſtant another impulſe will generate a velocity equal to the former. It will therefore move with the velocity 2, and at the third inſtant with the velocity 3, &c. for the preceding velocities are not in any reſpect diminiſhed or altered by the ſucceeding impulſes (21, Q). If then the impulſes are equal, and equidiſtant in time, the generated motion will be uniformly accelerated; and the velocity, which in this caſe may be conſidered as the motion, for the maſs of matter does not alter, will be in proportion to the time; that is, if the velocity in 5 inſtants be expreſſed by 5, that produced in 10 inſtants will be 10, &c. This holds good, let the number of impuſes in a given time be ever ſo great,

great. But the number muſt here be conſidered as infinite, for the force ceaſes not to act for the leaſt portion of time, and therefore the acceleration continues uniformly through every part of the motion.

E The ſpace deſcribed by an uniformly accelerated motion in a given time, may be conceived to be the ſum of an infinite number of ſpaces produced by a like number of uniformly increaſing velocities. Theſe ſpaces will be as the velocities by which they are deſcribed. Therefore, as the ſum of the numbers expreſſing the velocities in any given time is to the ſum of the numbers expreſſing the velocities in any other given time, ſo is the ſum of the ſpaces, or whole ſpace deſcribed in the firſt given time, to the ſum of the ſpaces, or whole ſpace deſcribed in the other given time. But the ſums of the velocities, for any terms of time taken from the beginning of the motion, are to each other as the ſquares of the times.

F For the times uniformly increaſing from the beginning, may be expreſſed by the natural ſeries of numbers, 1, 2, 3, 4, 5, &c. &c. The velocities, and alſo the correſpondent ſmall ſpaces, may be expreſſed by the ſame ſeries, becauſe they are both in the ſame ratio of the times. The whole time of deſcription, or the ſum of the inſtants, will be denoted by the number of terms, or, which is the ſame in this ſeries, by the laſt term. And the whole ſpace deſcribed, or ſum of the ſpaces, will be denoted by the ſum of the terms. Now, all arithmeticians teach, that twice the ſum of ſuch a ſeries is equal to the number of the terms added to unity, or 1, and multiplied by the laſt term. But, becauſe the inſtants contained in the given time have been aſſumed infinitely ſmall, the number of terms in the ſeries will be infinitely great, and conſequently the addition of unity to the number of terms will not cauſe any finite augmentation. Rejecting therefore the addition of unity, the double ſum of the terms will be equal to the number of terms multiplied by the laſt term, or, in this ſeries, to the laſt term multiplied by itſelf, that is, the ſquare of the laſt term; and becauſe halves are as their wholes, the ſum of the terms will be in the ſame ratio. But the ſum of the terms expreſſes the whole ſpace deſcribed, and the laſt term denotes either the whole time, or the laſt acquired velocity. Whence the ſpace deſcribed from the beginning of an uniformly accelerated motion is as the ſquare of the time, or as the ſquare of the laſt acquired velocity. Which was to be proved.

If the acceleration ceaſes at the end of any given time, the motion will become uniform with the laſt acquired velocity, and the ſpace deſcribed in an equal term of time will be double that which before was deſcribed from the beginning by the accelerated motion. For the ſpace deſcribed in any ſingle inſtant of the time by the laſt acquired velocity will be expreſſed by the laſt term of the juſt mentioned ſeries: and the whole ſpace deſcribed will be equal to this ſpace multiplied by the number of inſtants. But the time or number of inſtants being equal to the preceding time of acceleration, will be expreſſed by the number of terms, or, in this ſeries, by the laſt term. The ſpace deſcribed will be therefore expreſſed by the laſt term multiplied by itſelf; that is to ſay, it will be the double of the ſpace deſcribed by the accelerated motion in an equal time.

I When it is ſaid that a number denotes any magnitude, it is to be underſtood that the number is part of a ſeries, whoſe terms vary in a ratio always correſpondent, or equal to the ratio of the magnitudes denoted; that is to ſay, the ratio between any two terms of the ſeries is always equal to the ratio between the two magnitudes that correſpond to thoſe terms. In this ſenſe, any magnitudes, however unlike, may repreſent each other. Here follows a geometrical demonſtration of the foregoing propoſitions, in which lines are made uſe of inſtead of numbers.

K In the triangle A B C, fig. 5. let the equal diviſions A, 1, 2, 3, &c. on the ſide A B, repreſent equal parts of the time of an uniformly accelerated motion. Then the parallel lines, 1 d, 2 e, 3 f, &c. may repreſent the velocities at the ſeveral inſtants, 1, 2, 3, &c. for they are in proportion to the times A 1, A 2, A 3, &c. And in like manner for any other part of the time as A m, the velocity generated will be repreſented by its correſpondent ordinate m n. And the ſum of the ordinates correſponding with any part of the time will repreſent the ſum of the velocities. But the ſum of the ordinates, when taken indefinitely numerous, may be conceived to occupy the area contained between the ordinates of the firſt and laſt inſtants of the time. And theſe areas, when taken from the beginning A, are as the ſquares of the times A 1, A 2, A 3, &c. or of the velocities 1 d, 2 e, 3 f, &c. by the property of ſimilar figures. Therefore the ſums of the velocities, and conſequently the ſpaces deſcribed in any given terms of time taken from the beginning of an uniformly accelerated motion, are to each other as the ſquares of the times, or of the laſt acquired velocities. Hence it likewiſe appears, that the ſpaces deſcribed in equal ſucceſſive parts of time, are as the areas contained between A and 1 d, 1 d and 2 e, 2 e and 3 f, &c. which areas are to each other as the odd numbers 1, 3, 5, 7, 9, &c. as appears by inſpection from the number of equal and ſimilar ſmall triangles contained in each. L M

Again, ſuppoſe the motion at the end of the time A B to become uniform with the laſt acquired velocity B C. Complete the parallelogram B D E C, making B D equal to A B, and the ordinates 1 i, 2 k, 3 l, &c. will denote velocities, and conſequently ſpaces deſcribed. Their ſum will expreſs the whole ſpace deſcribed in a time equal to A B, and will be denoted by the area B D E C. But this area is equal to twice the area A B C. Conſequently the ſpace deſcribed by an uniform motion with the laſt acquired velocity, during a time equal to that of the acceleration from the beginning, will be double the ſpace deſcribed by the accelerated motion. N O

It will be readily evinced, that an uniformly retarded motion is exactly the reverſe of a motion uni- P formly accelerated. For ſuppoſe a conſtant force acting againſt a body in motion; as, for example, gravity acting againſt the motion of a body projected directly upwards, it will deſtroy an equal part of the initial velocity in an equal particle of time. Now, if theſe equal deductions be called unities, and be ſucceſſively taken from any number whatſoever, till the laſt remainder be nothing, it is evident that the ſeries of remainders will be the natural numbers 1, 2, 3, &c. in a reverſed order, and every thing that was proved of the times, ſpaces, and velocities (28, F. 30, K), or of the parts of the triangle ABC, fig. 5. will be true, mutatis mutandis, that is to ſay,

Q In the ſame body in motion, and retarded by a conſtant and equally acting force, the ſpaces deſcribed in coming to reſt, are as the ſquares of the
R initial velocities (29, G. 31, L), or, as the ſquares
S of the times during which they are deſcribed. And are equal to half the ſpaces, which in an equal time would have been deſcribed by their reſpective initial velocities uniformly continued.

CHAP.

CHAP. V.

OF THE PRODUCTION, COMMUNICATION, OR DESTRUCTION OF MOTION.

MOTION is produced, deſtroyed, or changed T
in a body, either by the impulſe, colliſion,
or ſtroke of another body in motion, or by the force
of attraction. Repulſion being immediately the con- U
trary to attraction, and not being perhaps ſufficiently
general and univerſal (6) to be admitted as a common property of body, need not be here conſidered.

We do not know whether the diſtinction be- V
tween impulſe and attraction be real, and exiſting in
the nature of things, or only relative to the imper-
fect ſtate of our knowledge. The moſt obſervable W
difference is, that impulſe is a force which acts from place to place, or, in other words, cannot be without motion: but attraction can exert itſelf even though no motion is produced.

To exemplify this, ſuppoſe two bodies to meet X
directly with equal quantities of motion; the effect of the ſtroke will be, that the whole motion will be deſtroyed, and the bodies will remain together. The forces will likewiſe be deſtroyed, and the bodies may be moved apart; each with the ſame facility as if the other did not exiſt. In this caſe, we have ſuppoſed no attraction to be exerted by them on each other. But let it now be ſuppoſed, that their motion, inſtead of being uniform, and the conſequence of their inertia, is produced by a mutual attraction. They come together, and the motion is deſtroyed as be-

fore. But the force of attraction, by which they were originally put in motion, remains, and is exerted in preſſing them againſt each other.

Y The ſmalleſt finite impulſe can overcome the greateſt finite preſſure. For, let any preſſure be ſuppoſed to produce acceleration, and the body, when in motion, will have more force than when it was merely preſſed. In its acceleration from reſt, it muſt paſs through every poſſible velocity leſs than the velocity laſt acquired. Let the impelling body have a momentum expreſſed by the product of its maſs into its velocity (19, L). Whatever product this may be, it is poſſible to aſſume a period of the acceleration of the body preſſed, in which its velocity ſhall be ſo ſmall, as that its product into its maſs ſhall be ſtill leſs. And it has already been ſaid, that the force of mere preſſure is yet leſs than this. Conſequently, it is leſs than that of the impulſe.

Z From this it is inferred, if two bodies, perfectly hard, or unyielding, were to ſtrike each other with any velocity, that they would be broken to pieces, provided the coheſion of their parts were leſs than infinite.
A But if the coheſion were infinite, it is preſumed that the communication or deſtruction of motion would
B be inſtantaneous. However, there are no ſuch bodies found in nature, and very conſiderable difficulties ariſe in the abſtract reaſoning concerning them.

C There appears to be the ſame relation between preſſure and momentum as between a line and a ſurface. In ſome reſpects, the firſt may be ſaid to generate the latter. Both are capable of increaſe or diminution, and yet no increaſe or diminution of the one can produce the other.

D That preſſure, which gravity cauſes bodies to exert againſt any obſtacle interpoſed between them and the earth, is called Weight. We have already taken notice of its effect when free to produce an uniform acceleration in falling bodies (26, A B C). If its power could be increaſed or diminiſhed, it would proportionally increaſe or diminiſh the momentaneous velocities and ſpaces, (28, E) and conſequently the whole ſpace paſſed through in a given time; that is to ſay, conſtant forces are as the ſpaces E paſſed through by acceleration in a given time, or as the laſt acquired velocities.

F When the effect of any retarding force is conſidered (31, P), the force will, in a given time, be as the whole ſpace deſcribed during the retardation, till the motion is deſtroyed, or as the initial velocity.

Let two accelerating forces be to each other in any ratio, the laſt acquired velocities will (35, E) be in the ſame ratio. Imagine the leſs motion to be continued till its laſt acquired velocity becomes equal to that of the other, and the whole time (27, D) will then be to the former time as the greater velocity to the leſs, or inverſely as the forces; that is, the times required to produce equal velocities G are inverſely as the accelerating forces. But the ſpaces deſcribed in equal times are as the forces (35, E). Whence the ſpaces deſcribed in any other

times

times will be as the forces and the ſquares of the times (29, G) jointly. But when equal velocities are produced, the ſquares of the times will be inverſely as the ſquares of the forces (35, G). Therefore the ſpaces in this caſe will be as the forces directly, and the ſquares of the forces inverſely, or

H inverſely as the forces; that is, the ſpaces paſſed through in producing equal final velocities are inverſely as the accelerating forces.

I When the effect of a retarding force is conſidered (31, P) the ſpaces paſſed through in deſtroying motion are inverſely as the retarding forces, when the initial velocities are equal.

K If a body be acted upon by a conſtant force, and its maſs be either increaſed or diminiſhed, without altering the force, the effect will be the ſame with reſpect to acceleration or retardation, as if the force, without changing the body, were diminiſhed or increaſed in the inverted ratio of the maſs. For the force being ſuppoſed invariable, will always produce or deſtroy the ſame quantity of motion (21, Q) in a given time. This quantity will be meaſured by the product of the maſs into the velocity (19, L). And that this product may continue unaltered, it is neceſſary that the velocity ſhould diminiſh in the ſame proportion as the maſs is increaſed, and the contrary.

L A body impinging with different velocities on tallow, clay, timber, and ſome other ſubſtances, penetrates to depths in the ſame ſubſtance which are as the ſquares of the initial velocities. Whence it follows, (32, Q) that theſe ſubſtances oppoſe a conſtant and invariable force of retardation againſt the motions of given bodies.

Suppoſe a body to impinge on an uniformly re-
ſiſting ſubſtance, if the initial velocity vary only, it
will penetrate to depths which are as the ſquares of
the velocities (32, Q). But if the maſs (not magni-
tude) vary only, the conſequence will be the ſame as
if the retarding force had varied in the inverted ratio
of the maſs (36, K). And the depths or ſpaces will
be inverſely as the retarding forces (36, I) or directly
as the maſſes: conſequently, if both the maſs and M
velocity vary, the depths will be in the compound
ratio of the maſſes and the ſquares of the velocities.

The diſpute concerning the meaſure of forces, N
which divided the philoſophical world for conſi-
derably more than half a century, was founded on a
partial conſideration of the effects of colliſion. The O
queſtion agitated was, whether the forces of bodies
in motion ought to be meaſured by the maſs of
matter multiplied by the velocity, or by the maſs
multiplied by the ſquare of the velocity. The for-
mer affirmation was called the old opinion, and the
latter the new opinion.

Neither of theſe opinions are ſufficiently general P
to apply to every caſe of motion, neither are they
repugnant to each other, as the contenders for each
inſiſted. The chief argument urged by the main- Q
tainers of the new opinion was, that ſpheres of equal
magnitude, but of different weights, being let fall
into tallow, from heights that were inverſely as the

weights, made pits of equal depths in the ſame. Now, ſaid the diſputants, equal cauſes are thoſe which produce equal effects; the forces of theſe bodies at their impact on the tallow muſt be equal, as being the cauſes of equal effects, namely, the pits in the ſame ſubſtance. But the ſquares of the velocities of the impacts are (29, G) as the heights from whence the bodies fall, or in this caſe inverſely as the weights of the bodies. Therefore the product of each weight or maſs into the ſquare of its velocity is equal to the product of the other weight into the ſquare of its velocity, when the pits, or, as it is affirmed, the forces, are equal.

R All this is true, when it is conſidered as a mere explanation of the meaning of the word force, which, if underſtood and applied in this ſenſe, will
S not be productive of error. But when the above is intended to ſerve as a proof that the action of a body in motion cannot be meaſured by the maſs multiplied into its velocity, it becomes neceſſary to obſerve, that the pit in the tallow being equal to another pit, does not prove that they were made by
T equal powers or forces. For powers cannot be ſaid to be equal, unleſs they produce equal effects in equal times; it being eaſy to imagine, that a weaker power continued for a longer time may produce an effect equal to that of a power of greater intenſity,
U though of leſs duration. And it is evident, that theſe pits are not deſcribed in equal times; for they are equal to half the ſpaces which would have been deſcribed with their reſpective initial velocities uni-

formly continued (32, s) in the reſpective times of deſcription. But the initial velocities would deſcribe ſuch equal ſpaces in times which are inverſely as the velocities themſelves. And it has been al- v ready ſeen, that the reſult of the preſent experiment is an eaſy conſequence of the properties of retarded motion, conſidered jointly with that definition which affirms the force or quantity of motion in a body to be as the product of the maſs into the velocity (37, M).

The conſideration that moving bodies penetrate w obſtacles to depths which are as the maſs of the body multiplied by the ſquare of its velocity, is of great uſe in almoſt every circumſtance of the kind. It follows from hence, that the depths to x which a body of given magnitude will penetrate into any ſubſtance, may be varied to infinity, without changing the quantity of motion. For the depths will always be greateſt when the velocity is greateſt (37, M); and the quantity of motion, or product of the maſs into the velocity, will not be changed, if the maſs be diminiſhed proportionally while the velocity is augmented, and the contrary.

Thus it is ſhewn, that a ſmall hammer, having y the ſame ſtriking ſurface and quantity of motion, will do more work at a blow than a large one. The driving of nails, or of piles into the earth, follows nearly the ſame law, though in the inſtance of the engine for driving piles by the fall of a weight, nothing (37, Q) would be gained by leſſening the weight and raiſing it higher: becauſe, from the property of the mechanical engines, the heights to which weights can be raiſed in a given time are inverſely as their weights.

Z When a body in motion ſtrikes another at reſt, it does not communicate the utmoſt quantity of motion to this laſt, until its whole action has been exerted, which, if it penetrates, is not until is has either penetrated to its utmoſt depth, or paſſed through.

A Hence a projectile may paſs through an obſtacle without communicating any conſiderable quantity of its motion, provided the obſtacle be conſideraby leſs thick than the depth to which the projectile muſt have penetrated by its whole efforts.

B This is exemplified in a piſtol-bullet ſhot againſt a door ſet open, through which it paſſes without communicating motion enough to overcome the friction of the hinges: whereas a large piece of lead, having the ſame momentum, inſtead of penetrating, carries the door before it, even though the ſtriking part be a prominence no larger than the bullet in the former caſe.

C The different effects of motion, according as the velocity is greater or leſs, is ſhewn likewiſe in the vulgar experiment of breaking a ſtick, whoſe ends reſt on two wine-glaſſes. Fig. 6. Let the two wine-glaſſes A, C, be filled, and let the ſtick A C be placed with its ends reſting on the two inner edges, as in the figure. Then, if a quick blow be ſtruck downwards, on the middle part B, the ſtick will be broken aſunder, without diſturbing either the glaſſes or their contents. In this experiment the two pieces A B, B C, are made to revolve on the points E and D, ſo that the points A and C riſe up inſtead of preſſing the edges of the glaſſes. On this account, if the blow at B be ſtruck underneath inſtead of of above the ſtick, the glaſſes will be broken.

That power or property by which a body re- D covers its figure, after it has been changed by any external action, is called elaſticity.

If a body ſtrike another at reſt, and one or both E of them yield inwards, or change their figure, the latter body will gradually, during the time of change, paſs through every poſſible velocity from reſt or nothing, to that which is expreſſed by dividing the quantity of motion in the ſtriking body by the ſum of the two bodies (19, L. 24, U); at which laſt inſtant both bodies will have the ſame velocity, and will proceed uniformly together, provided neither be elaſtic.

But if both bodies be perfectly elaſtic, they will F yield inwards, and the gradual change of velocity will obtain as before, till the inſtant of the utmoſt yielding, at which time both will have the ſame velocity. But the elaſticity being ſuppoſed perfect, both bodies will recover their figure with a force equal to that by which it was changed. The action of the elaſtic force being contrary to that of the former ſtroke, will cauſe the two bodies to ſeparate with the ſame relative velocity as they before came together. That is to ſay, the ſtriking body will loſe twice as much motion as it would have loſt by a ſimilar colliſion without elaſticity, and the body ſtruck will in like manner gain twice as much. Conſequently the ſtriking body, after the colliſion, will either proceed forward, remain at reſt, or be reflected back, according as its maſs is greater, equal to, or leſs than that of the body ſtruck.

H The experimental methods of illuſtrating the theorems relating to motion have been made, for the moſt part, on pendulous bodies, or on bodies let fall, either along inclined planes, or in the
I open air. Mr. John Whitehurſt, F. R. S. has invented an inſtrument for meaſuring the time a body employs in falling through a given ſpace. Its principal excellency conſiſts in meaſuring, to an accuracy, far beyond the reach of the ſenſes. The ſenſe can with difficulty divide a ſecond into twelve parts: but this inſtrument divides it into one hundred. A hand or index, connected with wheel-work, is made to revolve uniformly in a circle divided into one hundred equal parts, each revolution being performed in one ſecond. The regulator of the motion is a fly, whoſe leaves may be ſet ſo as to diſplace a greater or leſs quantity of air, accordingly as the inſtrument is required to go ſlower or faſter; or the ſame adjuſtment may be more accurately obtained by altering the weight that gives motion to the wheels. By the conſtruction of the inſtrument the body is let go at the beginning of a revolution, and at the end of its fall it ſtrikes another

another part of the mechaniſm, that inſtantly ſtops the hand, or index.

K Mr. G. Atwood, F. R. S. has invented an inſtrument for meaſuring the ſpaces paſſed through in a given time by a body in motion, whether that motion be accelerated, retarded, or uniform. It conſiſts of two cylindrical boxes, ſuſpended at the ends of a fine ſilk line that paſſes over a wheel or pulley. The axis of the pulley reſts on the circumferences of four other wheels, ſo that the effect of friction is ſcarcely ſenſible. If the two boxes be equally loaded, the weight of the one will counterpoiſe or deſtroy the effect of the gravity of the other. The two boxes with their contents, together with part of the pulley, may, in this caſe, be conſidered as compoſing a maſs void of gravity. Let any weight be added to one of the boxes, and as this weight is at liberty to move, it will be uniformly accelerated by the conſtant action of gravity (27, D): but by the conſtruction of the inſtrument, it muſt give an equal velocity to the whole maſs before mentioned, which, therefore, may be ſaid to be an increaſe of its maſs, while the moving force remains conſtant. The ſpaces paſſed through in a given time by the body will, therefore, be to the ſpaces it would have paſſed alone by the ſame action of gravity, (36, K. 35, E) as its own maſs or weight is to the whole maſs in motion. By this happy contrivance, the ſpaces paſſed through by acceleration or otherwiſe, in a given time, are rendered ſhort, and eaſy to be obſerved with preciſion,

ſion. For this intention there is a chronometer beating ſeconds added to the inſtrument, and a graduated rule near one of the boxes, with a moveable ſtage, to limit or terminate the motion *.

CHAP. VI.

OF THE ATTRACTION OF COHESION, AND OF SPECIFIC ATTRACTIONS.

L THE attraction of coheſion is that force by which bodies or their particles adhere to each other.

M It is demonſtrated, that if the forces by which the particles of bodies tend towards each other decreaſe in the proportion of the ſquares of their diſtances, the attractive force of two ſpheres compoſed of ſuch particles, will be governed by the ſame law; relation being had to the diſtances of their centers: and conſequently, it will not be ſenſibly greater when they are in contact, than when N they are at a ſmall diſtance from each other. But if the firſt mentioned forces decreaſe in the proportion of the cubes of their diſtances, or in any greater proportion, the latter will decreaſe after a much higher rate, and the bodies, when in con-

---

* This inſtrument, and its various uſes, are deſcribed at large in an excellent analytical treatiſe, by the inventor, intitled, " A Treatiſe on the rectilinear motion and rotation " of bodies."

tact, will attract each other very much more forcibly Y
than when ſeparated at the leaſt diſtance from
each other *.

The firſt of theſe attractions is gravity, as is O
evinced by its action on the planetary bodies, and
the latter appears to be the attraction of coheſion, P
for its force is vaſtly leſs at the leaſt diſtance, than
at the place of contact.

In conſequence of this law, ſeveral deductions
are made, which are found to agree with the phenomena
of this latter kind of attraction, as,

Thoſe particles which are poſſeſſed of large ſurfaces Q
of contact, adhere moſt ſtrongly together,
and form bodies which are called hard.

Thoſe particles which touch each other in few R
points, compoſe bodies which are ſoft or fluid, on
account of the ſmall force with which their parts
adhere together.

And hence probably may be explained the elaſticity S
of ſome bodies; for it ſeems to depend on
the coheſive force which reſtores the particles to
their firſt relative ſituation, when by any external
impulſe, they have been removed to a very ſmall
diſtance from each other.

Many diſcoveries remain for the induſtry of future T
philoſophers to make concerning this very
powerful agent in nature.

By this power the drops of all fluids aſſume a U
round form, and poliſhed plates of metal adhere

---

* Principia I. 76.—I. 86.

together with a prodigious force. This laſt is exem-
v plified by paring a ſmall part from each of two leaden bullets, and preſſing the ſurfaces together; in which caſe, with a ſurface of contact not exceeding the twentieth part of a ſquare inch, it will frequently require the force of 100lb. to ſeparate them.

w By this power alſo it is, that liquids riſe into the ſubſtance of bread, ſponge, and other porous bodies; and are ſuſtained in open capillary tubes a conſiderable height above the level. This height is in the reciprocal proportion of the diameters of the tubes.

x Two plain glaſs plates, fig. 7. touching each other at the line AB, and ſeparated at CD, by a ſmall obſtacle K, being placed in the veſſel of water EFGH, the water riſes between them to the line DIA, which is an hyperbola.

y Let two plain glaſs plates ABCD, fig. 8. be lightly moiſtened with oil of oranges, and placed one upon the other, ſo as to touch at the line AB, being kept ſeparate at CD, by the ſmall obſtacle L interpoſed. In this ſituation let them be placed in the horizontal box, EFGH, the part CD reſting on its bottom, and the other part towards AB, reſting on the upper end of the perpendicular ſcrew IK, which is fixed in the box for the purpoſe of raiſing the plates to any deſired angle of elevation. Then a drop of the above mentioned oil being applied in the opening CD, will be attracted by the two plates, and will proceed with an accelerated motion towards A B, if the plates are kept in an horizontal poſition. But if the end A B be raiſed by means of the ſcrew, to a conſiderable angle, the drop will remain ſuſpended in its courſe ſomewhere between C D and A B, ſuppoſe at N, and if the elevation be encreaſed, it will return towards C D, its weight overpowering the attraction of the plates.

Now, ſince the weight of the drop continues unaltered, it will not be difficult to find its tendency to return, or that part of its weight which is exerted in the inclined deſcent. For the proportion between that part and the whole, is as the height of the plane to its length, as will hereafter be ſhewn. And ſince the two powers, namely, the attraction by which the drop tends upwards, and that part of its weight which is exerted in the contrary direction, are equal when it remains ſuſpended, the meaſure or quantity of the one will expreſs the meaſure or quantity of the other. By theſe means it is eaſy to determine the attractive force; which is found to increaſe in the reciprocal proportion of the ſquares of the diſtances of the middle of the drop, from the end where the plates are in contact. That is, ſimply in a reciprocal proportion, becauſe the drop enlarges its ſurface as the ſpace becomes narrower; and again, ſimply in a reciprocal proportion, becauſe the attraction increaſes, the nearer the plates approach each other.

This coheſive attraction extends to an extremely ſmall diſtance from bodies, and where its power terminates, repulſion takes place, of which we ſhall ſubjoin a few inſtances.

A All hard bodies require a conſiderable force to bring them into contact, as appears by compreſſing a convex lens and plane glaſs together, which exhibit different appearances at the very point of ſuppoſed contact, according to the different degrees of compreſſion. This is likewiſe ſhewn from the paſſage of the electric matter through metallic chains: Of which more hereafter.

B When it rains on the ſurface of a veſſel of water, ſmall drops may frequently be ſeen running in all directions, which do not mix with the reſt of the water for ſeveral ſeconds.

C Hence likewiſe it is, that bodies ſpecifically heavier than any fluid may be made to ſwim on its ſurface; for, if by their repulſion a quantity of the fluid is diſplaced equal in weight to the ſolid, it will not ſink.

D Dry needles or thin plates of metal ſwim on water, and form cavities of a curve lined form, extending to a conſiderable diſtance from the body.

E Let ACB, fig. 9. repreſent the ſection of a veſſel of water, on whoſe ſurface AB is laid two circular plates of tinfoil, on each of which is placed a ſmall curtain ring, or ſome ſuch body, to encreaſe its weight, and cauſe it to ſink farther beneath the ſurface. By this means they may form two cavities about one-tenth of an inch deep, and extending half an inch every way from the circumferences of the plates. If they are brought within the diſtance of an inch from each other, they will ruſh together with an accelerated motion.

F Things remaining as in the laſt experiment, let D and E fig. 10. be two pieces of wet cork of the ſame dimenſions; the water then, by adhering to their ſides, may form a curve lined protuberance, extending about half an inch from their circumferences, and when they are brought within an inch of each other, they will ruſh together as before.

G Theſe appearances are eaſily accounted for, by conſidering, that action and reaction are equal. The plate of tinfoil, by its repulſion, acts on the water, and prevents its filling the cavity, and the water by its weight reacts on the plate; but as this reaction is the ſame on all ſides, no motion is produced. But when the two plates approach each other near enough to unite their repulſive actions, the weight of the water between them being diminiſhed by the depreſſion, its reaction is leſs than that which prevails on the oppoſite parts of the circumferences: conſequently they move in the direction of the greater preſſure, that is, towards each other.

H In the latter caſe, the reaction is in a contrary direction, being oppoſed to an attractive, inſtead of a repulſive force; for the reaction of the water endeavouring on all ſides to return to its level, produces no motion, becauſe its height is the ſame at equal diſtances all round. But when the two attractive powers are, by bringing the bodies nearer, made to act on the ſame water between them, it is raiſed higher above the common ſurface, and conſequently reacts more ſtrongly on that ſide; whence the bodies ruſh together, as in the former caſe. A depreſſion of the ſurface between the two corks will, by diminiſhing the quantity of water, occaſion them to recede from each other.

i It is not yet decided whether the attraction of coheſion, or the power by which bodies retain the aggregation of their parts, be one and the ſame with the attraction of combination, or chemical affinity. But as far as experiment has yet extended on this ſubject, there ſeems to be reaſon to believe that they are the effect of the ſame power.

k When the rational method of philoſophizing from obſervation and experiment was leſs known and eſteemed than it is at preſent, many objections were made to the admitting attraction as a general cauſe. Among others, it was ſaid to be a revival of the trifling philoſophy of occult cauſes. But nothing can be more inconſiſtent and abſurd than to compare that philoſophy, which deduces general laws from the obſervation of phenomena, with the compendious method that vanity has invented to diſguiſe or conceal human ignorance, by referring particular facts to occult cauſes. The laws of motion, the extenſion, the inertia, the reſiſtance, and the attraction of matter, are deduced in the ſame manner; and if the cauſes of theſe be occult or unknown, it is not that philoſophers are unwilling, but becauſe they have not yet been able to diſcover them, or becauſe ſome of them are, perhaps, ſo ſimple, as to be referable only to the will of the Creator. Nothing, however,

UNIV. OF MICH.

however, can be clearer, than that the exiſtence of a thing may be known, though its cauſe be latent; and it is enough that attraction really exiſts and acts according to eſtabliſhed laws, to juſtify philoſophers in admitting it in their explication of natural appearances.

BOOK I.

SECT. II.

Of bodies in motion.

CHAP. I.

OF THE MECHANICAL POWERS.

L WHEN two heavy bodies or weights are made by any contrivance to act againſt each other, ſo as mutually to prevent each other from being put into motion by gravity, they are ſaid to be in equilibrio. The ſame expreſſion is uſed with reſpect to other forces, which mutually prevent each other from producing motion.

M Any force may be compared with gravity, conſidered as a ſtandard. Weight is the action of gravity on a given maſs (35, D). Whatever therefore is proved concerning the weights of bodies will be true in like circumſtances of other forces.

N Weights are ſuppoſed to act in lines of direction parallel to each other. In fact, theſe lines are directed to the center of the earth, but the angle formed between any two of them within the ſpace occupied by a mechanical engine is ſo ſmall, that the largeſt and moſt accurate aſtronomical inſtruments are ſcarcely capable of exhibiting it.

o The ſimpleſt of thoſe inſtruments, by means of which weights or forces are made to act in oppoſition to each other, are uſually termed Mechanical Powers. Their names are, the Lever, the Axis and Wheel, the Pulley or Tackle, the inclined Plane, the Wedge, and the Screw.

p In the theoretical conſideration of theſe ſimple inſtruments, the parts they are compoſed of are imagined or ſuppoſed to poſſeſs no other properties than thoſe which conduce to the purpoſe of their conſtruction. Thus, they are all ſuppoſed to be without weight or inertia, and to move without friction. Many of theſe parts are taken to be mathematical lines, ſome perfectly inflexible, and others perfectly flexible, repreſenting ropes. And theſe ſuppoſitions are allowable, becauſe they imply nothing more than that the reaſonings relate only to perfect inſtruments, and it is conſequently no more to be urged that there are no perfect inſtruments, than that there are no perfect mathematical figures, at leaſt that ſenſe is able to diſcover or diſtinguiſh. For the difference between theory and practice is in ſome caſes inconſiderable, and may in general be allowed for without much difficulty, from the general principles of mechanics, when once eſtabliſhed.

CHAP. II.

OF THE LEVER.

THE lever is a moveable and inflexible line; Q acted upon by three forces, the middle one of which is contrary in direction to the other two.

One of theſe forces is uſually produced by the re- R action of a fixed body, called the fulcrum.

Let A C (fig. 11.) repreſent an horizontal lever at S reſt. At the point B, equidiſtant from A and C, is placed the fulcrum D, and at the extremities A and C are hung the equal weights E and F. Then the lever will continue at reſt, the weights E and F being in equilibrio. For it is evident, that if the line A C be moved on the fulcrum B, its extremities A and C will each be carried with equal velocities in the periphery of the ſame circle. And becauſe A C is horizontal, the actions of the weights E F will be in direction at right angles to its length: that is to ſay, they will act in the direction of tangents to the ſaid circle at the points A and C; or they will act in the direction of thoſe particles of the periphery, which may be imagined to coincide with the tangents. Each preſſure therefore tends to move the correſpondent extremity of the line A C in that very line of direction in which only it can move. Suppoſe the preſſure at C to be removed, and the whole preſſure at A will be employed in depreſſing the point A, or, which is neceſſarily in this caſe the ſame, in raiſing the point C: on account therefore of the equal velocities of the points A and C, the action of E at the point A will be the ſame as if it were exerted at C in the direction of the tangent C G. But again, ſuppoſe the equal weight F to be reſtored, and the point C will then be acted on by two equal and oppoſite forces, which, deſtroying each others effect, will not produce motion; conſequently the lever will continue at reſt.

T It is likewiſe evident, that if the radii A B and B C are not in a right line, the equal forces will nevertheleſs be in equilibrio, if they are applied in the directions of the tangents; thus, if B C be bent to the poſition B K, and the force F be there applied in the direction K F, the equilibrium will remain as before.

U If two contrary forces be applied to a lever at unequal diſtances from the fulcrum, they will equiponderate when the forces are to each other in the reciprocal proportion of their diſtances. For,

V Let A C (fig. 12.) repreſent a lever, whoſe radius A B is three times as long as B C. At A is ſuſpended the weight E of one pound, and at C is ſuſpended the weight F of three pounds. Then, I ſay, theſe weights will equiponderate. With the radius B A deſcribe the arc A K, interſecting C F at K. Draw the line B K, which may repreſent another arm to the lever; and it will be evidently of no conſequence, whether the thread C F be faſtened at C or K; conceive it therefore to be faſtened at K, and to act on the arm B K. Let A G repreſent the force of E, and K F, being three times as long, will repreſent the force of F. This force K F may be reſolved (23, T) into two others, K I in the direction of B K, and K H in the direction of the tangent, and their quantities are determined by drawing the lines F H and F I, parallel reſpectively to K I and K H. Now, K I has no effect in moving the arm B K. It is the force K H alone that tends to produce motion towards H. The triangles B C K and K H F are ſimilar, therefore, B K : B C :: K F : K H. But B K : B C, as 3 to 1, whence the force K H is $\frac{1}{3}$ of K F, as is likewiſe A G by the condition. Conſequently H K and A G are equal, and being applied at the end of equal arms A B, B K, will be in equilibrio (56, T), which was to be proved; and the concluſion will be the ſame, when the weights are to each other in any other ratio, provided the arms of the lever A B and B C be reciprocally in the ſame proportion.

By the reſolution of force it appears, that if two contrary forces be applied to a ſtrait lever at diſtantances from the fulcrum in the reciprocal proportion of their quantities, and in directions always parallel to each other; the lever will remain at reſt in any poſition.

For, let the forces, A E, C F (fig. 13.) be reſolved: A E into G E parallel, and G A perpendicular to A C; and C F into H F parallel, and C H perpendicular to A C; and the forces, which tend to produce motion, will in all poſitions be to each other in the ratio of the forces applied; i. e. A E : C F :: A G : C H, the triangles A G E and C H F being ſimilar.

Many

Y Many curious and uſeful effects may be produced by levers, whoſe arms are bent into an angle; but the limits of this work does not permit us to enlarge upon them.

Z It is evident, that all which has been ſaid concerning the lever is equally true, when the contrary forces are applied on the ſame ſide of the fulcrum.

A On the lever A B, (fig. 14.) if the weight E of one pound be applied at A, and the weight F of three pounds at C, ſo that their diſtances A B and C B, from the fulcrum B, may be as three to one, they will equiponderate; which is proved by applying the reaſoning at fig. 12. to the preſent figure.

B Since, of the three forces which act on a lever, the two which are applied at the extremes are always in a contrary direction to that which is applied in the ſpace between them; this laſt force will ſuſtain the effects of the other two: or, in other words, if the fulcrum be placed between the weights, it will be acted upon by, or will ſuſtain their ſum; but if the weights are on the ſame ſide of the fulcrum, it will be acted upon by their difference.

C On the principle of the lever are made, ſcales for weighing different quantities of various kinds of ſubſtances; the ſteelyard, which anſwers the ſame purpoſe by a ſingle weight, removed to different diſtances from the fulcrum on a graduated arm, according as the body to be weighed is more or leſs in quantity; and the bent lever balance, which, by the revolution of a fixed weight, increaſing in power as it aſcends in the arc of a circle, indicates the weight of the counterpoiſe. ABCDE (fig. 15.) is a bent lever, ſupported on its axis or fulcrum B in the pillar JH. At A is ſuſpended the ſcale E, and at C is affixed a weight; draw the horizontal line KG through the fulcrum, on which, from A and C, let fall the perpendiculars AK and CD; then, if BK and BD are reciprocally in proportion to the weights at A and C, they will be in equilibrio, but if not, the weight C will move along the arc FG, till that ratio is obtained. It is eaſy to graduate the plate FG ſo as to expreſs the weight in E by the poſition of C.

The beam of the common balance is a bent lever, E with equal arms. Its property of coming to reſt in an horizontal poſition, when the extremes are equally loaded is a conſequence of its being bent, or, which is the ſame thing, of its fulcrum being above the line, joining the two points on which the ſcales are ſuſpended. For it is evident, (fig. 16.) that there is but one poſition in which the lengths of the arms AB, BC, referred to the horizontal line DE, can be equal, and that is when the points A and C are on the ſame level.

Balances that move with very little friction on the F fulcrum, and are exactly equibrachial, are highly valued. But this laſt property is of leſs importance than is commonly imagined. For, if two balances be equally ſenſible, and one of them not equibrachial, it is certain, that if the ſtandard weight be placed in one of the ſcales of this laſt, and counterpoiſed, and the ſtandard weight be afterwards removed,

moved, any other body ſubſtituted in its place will have exactly the ſame maſs, if it be in equilibrio with the counterpoiſe. In fact, this is the beſt method of weighing, when great accuracy is required.

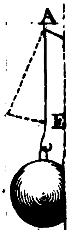
o On this principle alſo depends the motions of animals, the overturning or lifting great weights by means of iron levers called crows, the action of nutcrackers, pincers, and many other inſtruments of the ſame nature.

CHAP. III.

OF THE AXIS AND WHEEL, AND OF THE PULLEY OR TACKLE.

H THE axis and wheel may be conſidered as a lever, one of the forces being applied at the circumference of the axis, and the other at the circumference of the wheel, the central line of the axis being as it were the fulcrum. Fig. 17. is a perſpective view of the inſtrument, and fig. 18. is a ſection of the ſame at right angles to the axis. Then, if AB, the ſemidiameter of the axis, be to BC, the ſemidiameter of the wheel, reciprocally as the power E is to the power F, the firſt of which is applied in the direction of the tangent of the axis, and the other in the direction of the tangent of the wheel, they will be in equilibrio (56, U). For AC may be conceived to be a lever, whoſe

Axis
Fig. 1



wer o

UNIV. OF MICH.

whoſe fulcrum is B, and whoſe forces applied at A and C are in the reciprocal proportion of their diſtances.

I To this power may be referred the capſtan or crane, by which weights are raiſed, the winch and barrel, for drawing water out of wells, and numberleſs other machines on the ſame principle.

K The pulley is likewiſe explained on the principle of the lever. The line AC (fig. 19.) may be conceived to be a lever, whoſe arms AB and BC are equidiſtant from the fulcrum B. Conſequently the two equal powers E and F, applied in the directions of the tangents to the circle in which the extremities are moveable, will be in equilibrio (55, S). And the fulcrum B will ſuſtain both forces (58, B).

L But in fig. 20. the fulcrum is at C, therefore a given force at E will ſuſtain in equilibrio a double force at F, for in that proportion reciprocally are their diſtances from the fulcrum (56, V. 58, Z).
M Whence it appears, that, conſidering E as a force, and F as a weight to be raiſed, no increaſe of power is gained when the pulley is fixed, as in fig. 19. but that a double increaſe of power is gained when the pulley moves with the weight (fig. 20).

N A combination of pulleys is called a tackle, and a box containing one or more pulleys, is called a block.

Q ADB (fig. 21.) is a tackle compoſed of four pulleys; two of which are in the fixed block A, and the other two in the block B that moves with the

weight

P weight F. Now, becauſe the rope is equally ſtretched throughout, each lower pulley will be acted upon by an equal part of the weight: and, becauſe in each pulley that moves with the weight a double increaſe of power is gained; the force by which F may be ſuſtained will be equal to half the weight divided by the number of lower pulleys. That is, as twice the number of lower pulleys is to 1, ſo is the weight ſuſpended to the ſuſpending force.

Q But if the extremity C (fig. 22.) be affixed to the lower block, it will ſuſtain half as much as a pulley; conſequently the analogy will then be, as twice the number of lower pulleys, more 1 is to 1, ſo is the weight ſuſpended to the ſuſpending force.

It is for the moſt part more convenient to form tackles with blocks of the form exhibited in fig. 23.

R This reaſoning depends on the equal tenſion of the rope, and is therefore concluſive only when the tackle is wrought by a ſingle rope. In the ſyſtem of pulleys (fig. 24.) the power increaſes in a geometrical ſeries, whoſe common ratio is 2, and number of terms equal to the number of pulleys. Thus, if a force be applied at A, it will be acted upon by half the weight F; if at B, by $\frac{1}{4}$; if at C, by $\frac{1}{8}$; and if at D, by $\frac{1}{16}$; &c. The reaſon of which is evident from what has been already ſaid.

S It is evident, that in the compoſition of forces, the force produced is leſs than the ſum of the compounding forces; AD (fig. 3.) being always leſs than the ſum of AC and AB. On the contrary, in the reſolution of force, a gain of force is produced, which is exemplified in the following inſtance:

The rope E A C B F (fig. 25.) is paſſed over the pulleys A and B, and under the pulley C. Equal weights are ſuſpended at E and F, whoſe actions on C may be repreſented by the equal lines C J and C H. Theſe forces compounded give C D, which will expreſs the force exerted by the two weights E and F, tending to move C in the direction of the perpendicular, and is leſs than the ſum of C J and C H. Conſequently a weight G being applied at C, whoſe quantity is leſs than the whole quantity of E and F in the ſame proportion, will ſuſtain their effects, and remain in equilibrio. Therefore, if we conſider E and F, as producing by compoſition a force equal to G, a loſs of force enſues; and on the other hand, if G be conſidered as producing forces by reſolution equal to E and F, an increaſe of force is acquired. T

The quantity of this increaſe or diminution is readily determined thus. U

From J let fall the perpendicular J K upon C D, then C K will be the half of C D. And J C is half the ſum of J C and C H. Now, as the whole of that ſum is to C D, ſo is the ſum of the weights E and F to the weight G (for they reſpectively repreſent the forces of thoſe weights) and ſo is J C to C K. But J C is the ſecant of the angle formed between the rope A C and the perpendicular C D, the line C K being radius. Therefore, as the ſecant of the angle formed between one of the ropes, and the perpendicular is to radius, ſo is the ſum of the weights E and F to the weight G.

v Hence it follows, that the general deduction concerning pulleys and weights are only true when the ropes are parallel.

The pulley or tackle is of ſuch general utility, that it is needleſs to point out any particular inſtance.

CHAP. IV.

OF THE INCLINED PLANE, AND OF THE WEDGE.

w THE inclined plane has in its effects a near analogy to the lever. Let A B be an horizontal plane on which the weight E is placed, and let E D repreſent the force exerted by the weight. A B may alſo be conceived to act as the arm of a lever, whoſe fulcrum is A. Let this lever revolve on its fulcrum from B to C, then the weight E will be found at e, and will act on the plane A C with an oblique force e d, equal and parallel to E D. Reſolve e d into e b perpendicular, and b d parallel to A C, and the force e b will be deſtroyed by the reaction of the plane. With the other force b d, the weight will proceed with x an accelerated motion towards A. Whence it may be obſerved, that the inclined plane, acting againſt e in the manner of a lever, deſtroys that force which is exerted in the direction of the tangent of its line of motion, and that the acting force in this inſtrument is that which in treating of the lever

was rejected (56, v), as having no effect. The Y
force with which any weight on an inclined plane tends downwards in the direction of the plane, is to the weight itſelf, as bd to de. Or as eF to Ae, which is the ratio of the height of the plane to its length, becauſe the triangles bed and FeA are ſimilar. But eF is to Ae as the ſine of the angle the inclined plane makes with the horizon is to radius. (See fig. 26.) Therefore, as the ſaid ſine Z
is to radius, ſo is the force tending downwards in the direction of the plane to the weight. And becauſe radius is a conſtant quantity, the forces by which the ſame weight tends downwards in the directions of various planes will be as the ſines of their inclinations.

This inſtrument is not much uſed in its ſimple form.

If it be required to ſhew what force in the di- A
rection ep parallel to AB (fig. 27.) will ſuſtain the weight e in equilibrio. Set off em equal to bd, which will repreſent its force or tendency in the direction of the plane, and equal, but on the contrary ſide, ſet off en, which will repreſent the force that, applied in the oppoſite direction, will ſuſtain the weight in equilibrio. Draw np perpendicular to AC and ep parallel to AB, interſecting np in p; ep will be the force required; for it is compoſed of en and np, and np being perpendicular to the direction of the tendency of e avails nothing. Join pd and this laſt found force is to the whole weight of e, B
as pe to ed, or as eF to FA, which is the ratio of

the perpendicular height of the plane to its horizontal baſe, for the triangles ped and EFA are ſi-
C milar*. And ſince action and reaction are equal,
and in contrary directions (22, R), it is evident that the ſame force ep, which ſuſtains e on the fixed inclined plane CAB, applied in the contrary direction would, if the plane be ſuppoſed moveable in the direction of its baſe AB, and the body e fixed by the application of an obſtacle qr, ſuſtain the effort with which the ſaid body tends to impel the plane from e towards p.

D The wedge is compoſed of two inclined planes joined together at their common baſe, in the direction of which the power is impreſſed.

E Let ABC (fig. 28.) repreſent a wedge, whoſe vertex A is inſerted between the two bodies D and E, which being fixed in poſition, reſiſt in a certain degree any force which tends to ſeparate them. This reſiſtance uſually is, like the weight in the inclined plane, perpendicular to the baſe AF, and the power, or force employed to overcome it, is impreſſed as was juſt mentioned, in the direction of the ſaid baſe. Therefore, by the property of the inclined plane, the force required to keep one half CFA of the wedge in equilibrio with the preſſure of the body D, is to that preſſure as CF to FA. But as the preſſure on the other half of the wedge acts with equal effect, a double force will be required to preſerve the equilibrium, that is, a force as CB to FA. Or, in general terms; in any wedge, as the line CB, joining the two equal ſides AB and AC, is to the diſtance between the vertex A, and the middle point F of CB, ſo is the force impreſſed to the reſiſtance in D and E.

---

* The ſimilarity of theſe triangles not being obviouſly deducible is proved thus:

Prolong pe and dm till they meet in s; and the right angled triangle mse will be equal and ſimilar to the triangle npe, se being equal to ep.

The triangles sed, ped have the two ſides se, ep equal, the ſide ed common to both, and the included angles sed, ped are both right angles. Conſequently the triangles are equal and alike in all reſpects. Euclid I. 4.

But it is eaſily ſhewn, that the triangle sed is ſimilar to the triangle EFA, and ſo likewiſe muſt ped.

This inſtrument is commonly uſed in cleaving wood, and was formerly applied in engines for ſtamping watch-plates. The force impreſſed is commonly a blow, which is found to be much more effectual than a weight or preſſure. This difference is uſually accounted for, by ſuppoſing that the tremulous motion produced by the ſtroke, conſiderably diminiſhes the very great friction at the ſides. But there is no doubt, that it is chiefly referable to the principles that obtain when reſiſting bodies are penetrated (37, M).

All cutting inſtruments may be referred to the wedge. A chizel, or an axe, is a ſimple wedge. A ſaw is a number of chizels fixed in a line. A knife may be conſidered as a wedge when employed in ſplitting, but if attention be paid to the edge, it is found to be a fine ſaw, as is evident from the much greater effect all knives produce by a drawing ſtroke, than what would have followed from a direct action of the edge.

CHAP. V.

OF THE SCREW, AND OF MECHANICAL ENGINES IN GENERAL.

I THE Screw is compoſed of two parts, one of which is called the ſcrew, and conſiſts of a ſpiral protuberance, called the thread, which is wound or wrapt round a cylinder; and the other, called the nut, is perforated to the dimenſions of the cylinder, and in the internal cavity is cut a ſpiral groove adapted to receive the thread.

K Let A D G E (fig. 29.) repreſent a cylinder, and A B C any flexible ſubſtance of a thickneſs altogether inconſiderable or evaneſcent. Suppoſe A B C to be a triangle, having a right angle at A, and one of the legs A B containing the right angle to be applied to the cylinder in a line parallel to its axis. Imagine now the cylinder to turn on its axis, ſo that the triangle A B C may be rolled or wrapped cloſe on its ſurface. The lines B C, and all others, as I K, L R, parallel to it, will then be contiguous to, or coincident with, the peripheries of circles whoſe planes are all at right angles to the axis, and conſequently parallel to each other. But the line A C will become a curve A Q L M a N O P, &c.
L which is called an helix. This curve will always, or in every part, proceed from one towards the other

other end of the cylinder it enwraps, and will make equal angles with the generating circle of the cylinder. For any one of theſe angles, a k i, will be produced by the application of another angle A K I, always equal to the angle A C B.

Suppoſe the cylinder A D G E to be perpendicu- M
lar to the horizon, the lines B C and its parallels, together with all their correſpondent circles on the cylinder, will then become horizontal. Let the line A C now repreſent an inclined plane whoſe height is A B, and the helix being of the ſame length and height, and equally inclined to the horizon throughout, will not differ in mechanic effect from the inclined plane. That is to ſay, the N
tendency of a weight to deſcend on the inclined plane will be exactly the ſame as on the helix.

Let A L be the perpendicular diſtance between O
two adjacent threads. Draw the horizontal line L R interſecting A C in R. Then L R will be equal to the circumference of the cylinder, and A R will be equal to one revolution of the helix. But A R repreſents an inclined plane, equivalent in power to the helix. Every helix therefore is equivalent in P
power to an inclined plane, whoſe length is equal to one revolution of the helix, and height equal to the diſtance between two adjacent threads meaſured by a line parallel to the axis of the cylinder contained within the helix.

If the horizontal thickneſs of the nut be diſre- Q
garded, it will not differ from a weight to be ſuſtained on the helical plane. Conſequently it R

will be kept in equilibrio by an horizontal force, which is to that of the weight, as the perpendicular diſtance between two adjacent threads is to the circumference of the cylinder (69, P. 65, A, B).

S Or if the power be applied in the direction of the threads of the ſcrew, the equilibrio will be had when the power is to the weight as the perpendicular diſtance between the two adjacent threads is to the length of one thread of the ſcrew (69, P. 65, Y).

T But there are few, if any, inſtances where the ſcrew is uſed without the lever. If an arm EF (fig. 30.) be applied to move the nut, the weight and the power may be conſidered as acting upon a lever, whoſe fulcrum is at the axis of the cylinder.

U And, therefore, the proportions laſt found (R, S.) muſt be compounded with the ratio of the ſemidiameter of the cylinder to the diſtance of E from the axis of motion.

V It would be difficult to enumerate the very many uſes the ſcrew is applied to. It is extremely ſerviceable in compreſſing bodies together, as paper, &c. It is the principal organ in all ſtamping inſtruments for ſtriking coins, or making impreſſions on paper or cards, and is of vaſt utility to the philoſopher, by affording an eaſy method of meaſuring

W or ſubdividing ſmall ſpaces. A very ordinary ſcrew will divide an inch into five thouſand parts; but the fine hardened ſteel ſcrews that are applied to the limbs of aſtronomical inſtruments, will go much farther. This method will be readily underſtood from the contemplation of fig. 31.

On the rule EGHF is fixed the upright piece EF. X Through this piece the ſtem of the ſcrew AB paſſes, and is held by a collar, ſo that it may be moved on its axis without advancing or retiring in the direction of its length. The circular plate CD is fixed at right angles to the axis of the ſcrew which paſſes through its center. The piece IKML is adapted to ſlide lengthways on the rule. This piece has a ſquare aperture, acroſs which is ſtretched a fine wire OP at right angles to the graduated line on the rule that may be ſeen beginning at N. At IK, on the ſliding piece, is a raiſed part perforated helically to receive the ſcrew. Suppoſe the ſcrew to have fifty turns in the length of an inch, and the edge of the plate to be divided into 100 equal parts: ſuppoſe likewiſe, that the wire OP ſtands between two of the dividing lines, and that it is required to determine its diſtance from one of them. Turn the ſcrew, which of courſe will move the ſliding piece, and obſerve, with a magnifier, when the wire accurately covers the dividing line. Then the number of whole turns of the ſcrew employed in the operation will give as many fiftieth parts of an inch, and the odd diviſions of the plate CD will ſhew the number of hundredth parts of a turn, that is to ſay, hundredth parts of one-fiftieth of an inch, or five thouſandth parts of an inch.

The ſcrew has been applied with great ſucceſs in Y the diviſion of aſtronomical inſtruments *.

---

* By Mr. Ramſden, who has written a treatiſe on the ſubject.

Z It is eaſy to conceive, that when forces applied to mechanical inſtruments are in equilibrio, if the leaſt addition be made to one of them, it will preponderate and overcome the effort of the other. But the want of a perfect poliſh or ſmoothneſs in the parts of all inſtruments, and the rigidity of all ropes, which increaſes with the tenſion, are great impediments to motion, and in compounded engines are found to diminiſh about one-fourth of the effect of the power.

A The properties of all the mechanical powers depending, as has been ſhewn, on the laws of motion laid down in the beginning of this treatiſe, and the action, or tendency to produce motion, of each of the two forces, being applied in directions contrary to each other, the following general rule for finding the proportion of the forces in equilibrio on any machine will require no proof.

B If two oppoſite forces be applied to the extremes of any mechanical engine, in the direction of the lines, in which, by the conſtruction of the engine, the ſaid extremes would move; and the intenſities of the forces be to each other reciprocally as the velocities the extremes when put in motion would acquire in the ſame indefinitely ſmall time: then thoſe forces will be in equilibrio.

Suppoſe the forces to be weights, and the ſame may be expreſſed thus:

C If two weights applied to the extremes of any mechanical engine be to each other in the reciprocal proportion of the velocities reſolved into a perpendicular

UNIV. OF MICH.

pendicular direction, (rejecting the other part) which would be acquired by each when put in motion for the ſame indefinitely ſmall time, they will be in equilibrio.

D Whence it may be obſerved, that in all contrivances by which power is gained, a proportional loſs is ſuffered in time. If one man, by means of a tackle, can raiſe as much weight as ten men could by their unaſſiſted ſtrength, he will be ten times as long about it.

E It is convenience alone, and not any actual increaſe of force, which we obtain from mechanics. This may be illuſtrated by the following example:

F Suppoſe a man at the top of a houſe draws up ten weights, one at a time, by a ſingle rope, in ten minutes. Let him have a tackle of five lower pulleys, and he will draw up the whole ten at once with the ſame eaſe as he before raiſed up one; but in ten times the time, that is, in ten minutes. Thus we ſee the ſame work is performed in the ſame time, whether the tackle be uſed or not: but the convenience is, that if the whole ten weights be joined into one, they may be raiſed with the tackle, though it would be impoſſible to move them by the unaſſiſted ſtrength of one man.

G Or, ſuppoſe, inſtead of ten weights, a man draws ten buckets of water from the hold of a ſhip in ten minutes, and that the ſhip being leaky, admits an equal quantity in the ſame time. It is propoſed, that by means of a tackle, he ſhall raiſe a bucket ten times as capacious. With this aſſiſtance he performs it, but in as long a time as he employed to draw the ten, and therefore is as far from gaining on the water in the latter caſe as in the former.

H Since then, no real gain of force is acquired from mechanical contrivances, there is the greateſt reaſon to conclude, that a perpetual motion is not to be obtained. For in all inſtruments the friction of their parts and other reſiſtances continually deſtroy a part of the moving force, and at laſt put an end to the motion.

CHAP. VI.

OF THE CENTER OF GRAVITY.

I LET A B (fig. 32.) repreſent a long ſlender body of an inconſiderable thickneſs, which is attracted by another body in the direction of the ſmall parallel arrows, a b c d, &c. Then the motion of A B will be the ſum of the motions of all the parts ſituate between A and B. Interpoſe the pointed obſtacle C D, and A B may be conſidered as a lever; C being the fulcrum. Conſequently, if C be ſo placed that the parts between A and C may be in quantity and diſtance from the fulcrum equipollent to thoſe between C and B, the whole body will reſt in equilibrio on the point C. This point is called the center of gravity.

K The thickneſs of A B being inconſiderable, the point C may be eſteemed as the center of gravity, but is not ſo when the thickneſs is taken into the account. The foregoing illuſtration, beſides the advantage of its ſimplicity, may ſerve to ſhew that when we ſpeak of the whole attractive force of a body being collected in its center; as for example, the center of the earth, it is not to be imagined that any real power, or, as it were, magic force, is ſuppoſed to exiſt in that center. In the ſame manner the body A B ceaſes to move, not immediately becauſe its center of gravity is ſuſtained, as if the cauſe of motion exiſted in that center alone, but becauſe, by the property of the lever, the forces on the ſide C B are made to counteract and deſtroy thoſe on the ſide C A.

The center of gravity is defined to be a point L
about which all the parts of a body or bodies are in equilibrio.

Therefore, the center of gravity of two bodies, M
A and B (fig. 33.) will be a point C, in the right line that joins their centers of gravity, which is diſtant from the center of each body in the reciprocal proportion of their maſſes (56, U); that is, A C : C B : : B : A. And the center of gravity of three bodies, ſuppoſe A, B and E, will be found at D in the line C E, which joins the center of gravity of E with the point C. C D being to D E reciprocally as the ſum of the maſſes of A and B is to the maſs of E. For it is eaſily proved, ſuppoſing the lines to be levers, that the bodies A and B will equilibrate on the point C, which, as the fulcrum, will ſuſtain both their forces (58, B); and alſo, that the body E will equilibrate with the force ſuſtained at C; D being

fulcrum. In this manner the center of gravity of any ſyſtem of bodies may be found.

N Though the point called the center of gravity is defined from the univerſal property, gravity, yet, it may be as well defined from the inertia of matter; for the point C (fig. 32.) is the center of inertia, or the point at which an impulſe will move the body without producing rotation; or, if the body were in motion in a right line, without rotation, the point C is the only point at which the oppoſition of a fixed obſtacle will deſtroy the whole motion at once; but our intended conciſeneſs forbids the elucidation of rotatory motions.

O If two bodies move uniformly in right lines, their common center of gravity will either be at reſt, or will move uniformly in a right line; and the ſame is likewiſe true of the center of gravity of three bodies, for the center C (fig. 33.) of any two of them may be conſidered as one body. Therefore, if C and E be in motion, the common center D will either be at reſt, or will move uniformly in a right line. And the ſame may thus be ſhewn of any number of bodies *.

P The common center of gravity of two or more bodies does not change its ſtate of motion or reſt from the mutual actions of the bodies upon each other; and therefore, the common center of gravity of all bodies mutually acting upon each other, is either at reſt, or moves uniformly in a right line,

---

* Principia, in Corol. 4. ad 3. Legem motus.

actions and impediments from without being excluded. For,

If the bodies A and B (fig. 33.) act upon each Q
other, the motion produced in each will be equal, (22, R) and the ratio of C A to C B will conſequently remain the ſame, whether they approach to, or recede from each other. The ſtate of C will not therefore be changed by their mutual actions. If the third body E be added to the ſyſtem, the center D, for the ſame reaſon, will not be changed, as to its ſtate of motion or reſt, whether E acts upon C or not: and the ſame may be proved of any number of bodies.

Since then the ſtate of the center of gravity of R
any ſyſtem of bodies, as to reſt, or uniform direct motion, is not affected either by the motions or mutual actions of the bodies of which it is compoſed, external actions or impediments being excluded, it is plain that the ſame law holds good in the motion of a ſyſtem of bodies as is obſerved by a ſingle body. For the progreſſive motion of a ſingle body, or of a ſyſtem of bodies, muſt be eſtimated by the motion of the center of gravity.

Hence it is that the center of gravity of the S
earth is not affected by the motions on its ſurface, or in its bowels. When a projectile, a cannon ball, for inſtance, is thrown upwards, the projecting force reacting on the earth, cauſes it to move in the contrary direction; but as the motions are equal, the center of gravity remains the ſame.

T The motions and actions of bodies upon each other in a ſpace that is carried uniformly forward, are the ſame as if that ſpace were at reſt.

U For the motions and actions of bodies upon each
other depend on their relative motion, the velocity
of which is the ſum of their abſolute velocities, when
they are moved in oppoſite directions, or their dif-
ference when they move in the ſame direction. And
this ſum or difference is not altered by an equal
velocity impreſſed on all the bodies in the ſame or
a parallel direction, as in the preſent caſe: ſince,
when two bodies move in contrary directions, in a
ſpace carried uniformly forward, the velocity added
to that body, with whoſe motion the [*] motion of
the ſpace conſpires, is exactly equal to the velocity
deſtroyed in the other body, whoſe motion is
oppoſed by that of the ſpace; and when the bodies
move in the ſame direction, an equal velocity be-
ing added to, or deſtroyed in both, the difference is
V likewiſe unaltered. This is likewiſe confirmed by
daily experience; motions performed on board a
ſhip under ſail are the ſame as if the ſhip were at
anchor; except ſo far as they may be diſturbed by
the irregular toſſing of the waves, which affects
them ſucceſſively, as much in one direction as
another. A fleet of ſhips carried by an uniform

---

[*] Space being in its own nature immoveable, the expreſſion is here improper; but it conveys a clear idea of the propoſition in conciſe terms, though we can form no idea of bodies included in a ſpace being acted upon by that ſpace. The ſpace here mentioned is merely ideal, may be called relative, and is defined to be a moveable dimenſion.

current, either preſerve the ſame relative poſitions, or approach to, or recede from, each other in the ſame manner as they would if no ſuch current exiſted. And the motions of bodies at the ſurface of the earth are no otherwiſe affected by its revolution on its axis than as the revolution is not rectilinear, the effects of which, though conſiderable, are not enough ſo to fall under common obſervation.

This propoſition is likewiſe true, if the motion w of the ſpace be uniformly accelerated, or, which is the ſame thing, if all the bodies be conſtantly acted upon by parallel forces which act equally, according to their maſſes, on each of them.

For ſuch forces will cauſe all the bodies to move x with the ſame acceleration, and to deſcribe equal ſpaces in the ſame direction with each other. They will not therefore change their relative motions or ſituations.

CHAP. VII.

OF PENDULUMS.

Y THE bodies ſpoken of in the preſent chapter
are ſuppoſed to move without rotation,
friction, or reſiſtance from the air, or any other medium;
neither are the magnitudes of the bodies
brought into conſideration.

It has already been ſhewn (65, Y), that the force
of a body to deſcend along an inclined plane is to
the whole force of its gravity as the height of the
Z plane to its length. If the body be at liberty to
deſcend on the plane, the firſt mentioned force will,
by its conſtant and equal action (27, D) produce
an uniform acceleration.

The ſpaces deſcribed from the beginning on a
given inclined plane are (29, G) as the ſquares of
A the times; that is to ſay, the times of deſcription
of inclined planes of the ſame inclination are as the
ſquare roots of their lengths.

B The final velocities (36, H) of accelerated motions
being equal when the forces are inverſely as
the ſpaces paſſed through, and the length of an inclined
plane being to its height in this ſame ratio
of the forces, by which a body would deſcend along
the plane, or fall freely through its height, it follows
C that the final velocity acquired by a body that
deſcends along ſuch a plane is equal to the final
velocity it would acquire by falling freely through
D its height. Hence alſo the final velocity is always
equal

equal when a body has fallen through an inclined plane of a given height, whatever may be its length.

E The times of acquiring this given velocity, or of paſſing over the whole lengths, will be inverſely as the forces (35, G); that is to ſay, directly as the lengths when the heights are equal (65, Y).

F Bodies that deſcend from a given height always acquire the ſame final velocity, whether they deſcend along a ſingle plane or many. Let E G (fig. 34.) repreſent an horizontal, and A G a perpendicular line; and ſuppoſe a body to deſcend along the inclined planes A B, B C, C D, D E; continue A B to F in the line E G, and draw the lines B K, C I, D H, parallel to the horizon. The body after paſſing through A B and B C will have acquired a velocity equal to the velocity it would have acquired ſimply by deſcending along B C or (80, D) along B L, added to the velocity it had at B: therefore the velocity at C, after paſſing through the planes A B, B C, is the ſame as would have been acquired by deſcending from the ſame height A I through a ſingle plane A L. The ſame reaſoning may be extended to prove, that when the body has arrived at D, it will have the ſame velocity as it would have acquired by deſcending in one plane A M of the ſame height A H: and ſo forth for any number of planes whatever.

Since the planes along which a body may paſs, in deſcending from a given height, are not limited either in number or magnitude, we may aſſume them to be indefinitely ſmall, and indefinitely numerous.

They may then be conceived to form a curve, and
G it will follow, that the laſt acquired velocity of a
body that deſcends by a gravity from a given height
along a ſurface either plane, polygonal or curved,
is always the ſame, and is equal to that velocity
it would have acquired by deſcending from the
ſame height by the action of its gravity in free
ſpace.

H On planes of the ſame inclination the times of deſcent from the beginning are as the ſquare roots of the lengths of the planes (80, Z. 29, G).

I If a body deſcend along any number of inclined
planes, AB, BC, CD, DE, (fig. 34.) and another body
deſcend along a like number of planes, NO, OP, PQ,
QR, (fig. 35.) having reſpectively the ſame incli-
nation and proportional lengths, namely, NO to
AB as OP to BC, and as PQ to CD, &c. then the
times of deſcent will follow the ſame law as would
have obtained if each had paſſed down a ſingle
K plane of the ſame inclination: that is to ſay, they
will be as the ſquare roots of the lengths paſſed
over. For, if the bodies were each to deſcend ſin-
gly along any two correſpondent planes AB and
NO, the times would be as the ſquare roots of thoſe
lengths (82, H): and the final velocities would be
in the ſame ratio (29, G). The ſame will be true
of the planes BC and OP. But now, ſuppoſe each
body to have deſcended through two planes AB,
BC, and NO, OP, then the time of deſcent through
BC will be leſs than it would have been in pro-
portion as the velocity it has at B is greater, namely,

in proportion to the ſquare root of A B (29, G). And in like manner the time of paſſing ſingly through O P will be diminiſhed in proportion to the increaſe of velocity gained at O, or in proportion to the ſquare root of N O. But A B is to B C as N O to O P, and therefore the times of paſſing through B C and O P are diminiſhed in proportion to their magnitudes, and muſt continue to have the ſame ratio as before. Again, ſince N O is to A B as O P to B C, the ſum of N O and O P, namely, N P, will be to the ſum of A B and B C, namely, A C, in the ſame ratio; and the ſum of the ſquare roots of N O and O P, namely, the ſquare root of N P, will be to the ſum of the ſquare roots of A B, B C, namely, the ſquare root of A C, as the ſquare root of N O to the ſquare root of A B, or as that of O P to that of B C. And ſince it has been ſhewn, that the times of paſſing over the planes N O, O P, and A B, B C, are reſpectively proportional to the ſquare roots of their lengths, the whole times muſt be proportional to the ſquare roots of their ſums, or whole length, which was to be proved: and the ſame reaſoning will apply to any number of planes.

If the planes be indefinitely ſhort and numerous, L they may be conceived to form a curve, and a ſimilar aſſemblage of planes reſpectively, in proportion to the former, will form a ſimilar curve. The foregoing arguments will then prove, that bodies deſcending along ſimilar curve ſurfaces deſcribe them in times which are as the ſquare roots of the lengths of the curves.

M If a body be urged towards a given point, by a force which is proportional to the diſtance of the body from the point, it will always arrive at the point in the ſame time, whatever the diſtance may be. Let I (fig. 36.) be the point; and ſuppoſe two bodies to be let fall from any two points, A and R; join A R, A I, and R I; and imagine A I to be divided into an indefinitely great number of equal parts, which, conſequently, will each be indefinitely ſmall. Through each point of diviſion in A I, imagine the lines B Q, C P, D O, &c. to be drawn parallel to A R, and they will divide the diſtance R I into an equal number of indefinitely ſmall parts. Any two intervals, A B and R Q, will be in proportion to the whole lines A I, R I, of which they are like parts. Now, the force may be eſteemed to be invariable or conſtant, while the body paſſes over the interval A B, becauſe the diſtance is not definitely leſs, during that time: and the ſame may be ſaid of the interval R Q; that is to ſay, the forces being as the diſtances A I, R I, will be alſo as the intervals or ſpaces A B, R Q, paſſed through: and the times of paſſing thoſe intervals will be equal (35, E). By the ſame argument the ſucceeding intervals B C, Q P, would have been paſſed over in equal times, if the motions had commenced at B and Q. But in the preſent caſe, the velocities already acquired in B and Q are (35, E) proportional to the ſpaces A B and B Q, and conſequently are ſuch as would, without any other action, carry the bodies over the ſpaces B C and Q P, in equal times. It is therefore evident, that the bodies are carried over the ſecond ſpaces B C and Q P, by means of actions or forces which would have carried them reſpectively through thoſe ſpaces in the ſame time, increaſed by acquired velocities that would likewiſe have carried them through the ſame in equal times. The acquired velocities in B and Q muſt therefore ſubduct equally from the times in which B C and Q P would have been otherwiſe deſcribed, and the remaining or actual times of deſcription will be equal. This reaſoning will extend to prove, that all the other correſpondent intervals, C D and P O, D E and O N, &c. are reſpectively paſſed over in equal times; or, in other words, the whole lines A I and R I will be paſſed over in equal times; which was to be ſhewn.

N If a number of indefinitely ſhort inclined planes be joined together, and the ſine of the inclination of any plane to the horizon (65, Z) be as its diſtance from the loweſt plane, meaſured along the planes, the tendency of a body to deſcend on them will be as its diſtance thus taken: conſequently (84, M) a body will, by paſſing along them from any diſtance, arrive at the loweſt point in the ſame time. O The line in which the deſcent is made may be conceived to be a curve, becauſe no part of any definite magnitude lies in the ſame right line. This curve is termed a cycloid.

P Every thing that has been proved of the accelerated deſcent of bodies along inclined planes will hold good, mutatis mutandis (31, P), when bodies are retarded in their aſcent along the ſame, or congruous planes.

Q A pendulous body oſcillates by the ſame laws as it would move on an inclined ſurface of the ſame figure as the curve it deſcribes. For the reaction of the ſtring or rod is exerted againſt, and deſtroys the very force that ſuch a plane would deſtroy.

R The pendulums of clocks uſually vibrate in the arcs of circles. It has formerly been thought an advantage to make them vibrate in the arcs of cycloids; but the difficulties that attend the practical application are ſuch, that there is good reaſon to think that they produce greater errors in the admeaſurement of time than thoſe they are intended to remedy*. For this reaſon, we ſhall here only explain the properties of bodies vibrating in circular arcs.

S Let B (fig. 37) be a body pendulous from A, and moveable in the arc BFE, whoſe loweſt point is F: draw the line AFC, on which from B let fall the perpendicular BD. This laſt line BD will be horizontal. From B draw BC at right angles to AB, and conſequently touching the arc BFE. The body at B will then be urged towards F by its gravity in the ſame manner as if it reſted on an inclined plane BC, making the angle DBC with the horizon. Which angle, by reaſon of the ſimilarity of the triangles DBC, DAF, is equal to the angle DAB, or the inclination

---

[*] The excellent Huyghens has explained the vibrations of pendulous bodies in his Treatiſe de Horologio Oſcillatorio; and the ſame thing is performed more generally in the Principia I, § 10.

clination of the pendulum-rod or ſtring with the perpendicular. But the force at B is as the ſine of this angle (65, z). Now, in very ſmall angles, the ſines, and conſequently the forces, are nearly proportional to the ſubtending arcs to be paſſed
over: therefore, (85, N) all the circular vibrations T
of the ſame pendulum are nearly equal when the arcs of vibration are ſmall.

But, in fact, becauſe the arcs increaſe faſter than U
the ſines, the leſs vibrations of the ſame pendulum are performed in leſs times.

The times of the vibrations of pendulums that V
deſcribe ſimilar arcs of circles, or, which is in fact the ſame, have equal angles of vibration, are (83, L) as the ſquare roots of the lengths of the arcs: and, becauſe the ſimilar arcs are as the radii they are
deſcribed with, the times will be as the ſquare roots W
of the lengths of the pendulums. This propoſition X
may be affirmed likewiſe of all angles of vibration (87, T) when they are ſmall.

Thus far, in our reaſoning, we have ſuppoſed the Y
force of gravity to be invariable in a given body; but if it be ſuppoſed to vary, its effects will vary in the ſame ratio (35, D, E); and the lengths of ſimilar arcs deſcribed by a pendulous body in equal times will be directly as the forces of gravitation that urge them. Now, the lengths of the arcs are as their radii, that is to ſay, the lengths of pendulums, vibrating through ſmall arcs (87, X) and meaſuring equal times, are as the gravitating forces.

Z If the ball B (fig. 37.) of a pendulum were let go from any point remote from the loweſt point F, it would deſcend to F with an accelerated motion through the arc B F, and if its motion were without all reſiſtance or impediment, it would, by a retarded motion preciſely ſimilar to its former acceleration, aſcend through an arc F E, equal to B F. From the point E it would again deſcend by acceleration to F, and again riſe to B by paſſing through the arc F B in the ſame manner as it before aſcended through the arc F E: the oſcillation would thus continue for ever, the angles and the times of all the vibration being equal. But this cannot be; for there is no avoiding a certain degree of friction at A, and the ball B ſtriking, and giving motion to the parts of the air oppoſed to its courſe, muſt itſelf loſe as much motion as it communicates, (22, R) by which means the motion muſt continually decay, and at laſt become inſenſible.

A The pendulums of clocks are maintained in their motion by the action of the wheels, which are driven round by means of a weight or ſpring. Let E G F (fig. 38.) repreſent the ſwing-wheel of a clock that is urged to revolve in the direction E G F; let C and D repreſent the pallets moveable on an axis at A, and ſo connected with the pendulum A B, that they are made to vibrate along with it. Suppoſe the ball to vibrate from R to B, one of the teeth of the wheel reſting againſt the pallet C, whoſe figure may be ſeen enlarged at I K L. When the pendulum returns towards Q, the pallet C is drawn out, the wheel preſſing firſt along the plane I K, and afterwards on the inclined plane K L; by its action on which laſt, it puſhes the pallet, or aſſiſts its motion, till at length the tooth ſlips off the point L. During this time the pallet D, whoſe figure is ſeen at P N O, is carried in between the teeth on the other ſide of the wheel; that when the wheel eſcapes the pallet C, another tooth drops on the plane N P of the pallet D. The returning vibration again draws out the pallet D, the tooth of the wheel aſſiſting its motion by preſſing along the inclined plane N O till it eſcapes at O. At this inſtant the pallet C has acquired its original ſituation, and therefore receives the adjacent tooth: and the whole proceeds as before. This is a very ſimple and good eſcapement. It is called the dead-beat eſcapement, becauſe the ſecond hand in clocks of this conſtruction falls with a dead ſtroke on the diviſion line of the dial, and does not recoil, but remains motionleſs during that part of the vibration in which the tooth of the crown-wheel reſts on the plane I K, or P N of the pallet.

If the reſiſtances ariſing from the friction at the moving parts, and from the motion communicated to the air, were always the ſame, and the clock were urged by a weight, the action of the ſwing-wheel on the pallets would be always the ſame at a given place, in conſequence of which, the figure of all the parts being ſuppoſed invariable, the arc of vibration would be conſtantly of the ſame magnitude, namely, ſuch as, that the motion

tion loſt by the reſiſtances oppoſed to the pendulum ſhould be accurately equal to the motion communicated by the pallets, and the times would be equal; that is to ſay, the clock would be perfect, and would meaſure time accurately. But theſe conditions are not eaſily obtained. It is not found, that the variation in the reſiſtance of the air, ariſing from its changes of denſity, occaſions any ſenſible irregularity in clocks. The moſt conſiderable irregularities in the movement ariſe from the tenacity of the oil applied to the moving parts. For the oil is leſs fluid in cold than in hot weather; and when it is leſs fluid, a greater quantity of the maintaining power muſt be loſt in overcoming its rigidity: whence it muſt happen, that the teeth of the crown-wheel will in that caſe act leſs forcibly on the pallets, and the vibration will be leſs. If the pendulum be ſuſpended on an axis, this cauſe, together with the conſtant wear, is very noxious; but this defect is remedied by ſuſpending it by a ſtrait flexible ſpring. And in addition to this, there have not yet been found any materials which do not expand by heat. The pendulum-rod therefore is longer in hot than cold weather, and the clock conſequently goes ſlower (87, w).

C It has not been determined from experiment and obſervation, how far theſe cauſes reſpectively affect the regularity of the clock's going. There are many good inventions for obviating their effects.

D The irregular action of the maintaining power, in conſequence of its giving motion to the whole train of wheels, is rendered of no conſequence by means of the eſcapements, which are called detached or free eſcapements. For in theſe the impetus of the ſwing-wheel is not ſuffered to act on the pendulum, but is employed in raiſing a ſmall weight in each vibration, which, by its fall, always gives the ſame impulſe to the pendulum.

The expanſion or contraction of deal-wood lengthways, by change of temperature, is ſo ſmall, that it is found to make very good pendulum-rods. The wood called ſapadillo is ſaid to be ſtill better. There is reaſon to believe, that the previous baking, varniſhing, gilding, or ſoaking, of theſe woods in any melted matter, only tends to impair the property that renders them valuable. They ſhould be ſimply rubbed on the outſide with wax and a cloth. In pendulums of this conſtruction the error is greatly diminiſhed, but not taken away: but there are a conſiderable number of ingenious contrivances for entirely removing it. E

The combination of metallic bars in the gridiron pendulum ſeems to be the moſt ſimple and effectual contrivance for this purpoſe, and is therefore the only one we ſhall here deſcribe. From the point of ſuſpenſion A (fig. 39.) proceeds a ſmall flexible ſpring, by the alternate flexure of which the vibration is allowed to be made. The lower part of the ſpring is fixed to the frame B C, out of which proceed five equal cylindrical bars of metal. The two outer bars are ſteel, as is likewiſe the middle bar: the other two are a compoſition of zink and ſilver. F ſilver. The two outer bars are faſtened in the piece B C by means of pins, but the three inner ones paſs looſely into holes in the ſame piece, without being faſtened at all. The three inner bars paſs through a piece D E, near their upper ends, in which they are all faſtened by pins. At the lower extremities of the bars is another croſs piece F G, in which all the bars are faſtened by pins, except the middle bar that paſſes freely through, and carries the ball or lens H.

G The conſideration on which this conſtruction depends is, that a given increaſe of temperature will cauſe the bars of zink and ſilver to expand about twice as much in their linear dimenſions as the bars of ſteel. To ſimplify our explanation, let us attend only to the ball H, without regarding the weight of the other parts. It will then only be required, that the diſtance between A and the ball H ſhould continue unaltered in every change of temperature, which is accompliſhed thus. Imagine the outer bars of ſteel to expand by heat, and they will ſuffer the frame F G to deſcend; the middle ſteel-bar will likewiſe expand in the ſame ratio of its length; and the ball H would conſequently be removed farther from the point of ſuſpenſion A if no other parts of the apparatus was affected by the heat: but the ſame heat expands the bars of zink and ſilver much more in proportion to their lengths, and therefore the adjuſtment may be ſo made as that the expanſion of theſe ſhorter bars may be equal to that of the longer ones of ſteel;

that is to ſay, the expanſion of theſe two bars may be ſuch as to remove the croſs piece D E farther from F G, ſo as to raiſe the ball H upwards through a ſpace exactly equal to the quantity of the expanſion of the central, and the two outer bars. Whence the ball H will always be kept at the ſame diſtance from A.

It may be obſerved, that the two outer ſteel-bars H anſwer the purpoſe of a ſingle bar with regard to the expanſion, as do likewiſe the two next adjacent to the middle bar. Theſe bars, namely, two of ſteel and one of zink and ſilver, would have been ſufficient in theory, but the neceſſity of binding them together in that caſe produces a degree of friction of the parts that is, not without reaſon, thought to have a bad effect, by cauſing the action to take place by ſtarts. The adjuſtment of the contrary expanſions is made by pinning the piece D E at the various diſtances from F G, by which means the ratio of the length of ſteel to that of the zink and ſilver may be altered in any convenient degree: and this adjuſtment being made from actual obſervation of the clock's going, it is evident, that no practical inconvenience can reſult from our gratuitous ſuppoſition of all the parts, except H, being without weight.

CHAP.

CHAP. VIII.

OF THE MOTION OF A BODY, WHICH IS ACTED UPON BY A CENTRIPETAL FORCE.

I IF a body at A (fig. 49.) be carried with an uniform direct motion in a given line A E, and rays be drawn from the equidiſtant points A, B, C, D, E, to any point L, without the line A K, the areas A L B, B L C, C L D, D L E, &c. will be equal to each other*.

K And theſe areas which are deſcribed in equal times, will not be altered by any centripetal force acting on the body A, and impelling it towards L. For,

L Suppoſe the body A to deſcribe the equal ſpaces A B, B C, C D, and conſequently with reſpect to the point L, the equal areas A L B, B L C, C L D, in equal times. Let a centripetal force be impreſſed at D, which ſingly would cauſe it to deſcribe the ſpace D d in the ſame time as D E, which is equal to D C, &c. Complete the parallelogram D d e E, and (23, s) at the end of the time the body will be found at e; having deſcribed the diagonal D e. The triangle D e L will then be equal to D E L, becauſe both ſtand on the ſame baſe D L, and between the parallels D L and E e. Continue e F equal to D e, and the area e F L will be equal to D e L, for the ſame reaſon; e F repreſenting the ſpace which would be deſcribed in the ſame time as D e, if no new impulſe were given at e. Let a force e m be impreſſed at e, and by the ſame proceſs it is proved, that e f L is equal to e F L. The like may be proved of the triangles f g L, g h L, &c.

---

* This reaſoning depends on that well-known propoſition (Euclid I. 38.) that triangles conſtituted upon equal baſes, and between the ſame parallels are equal to one another.

Since therefore any ſingle impulſe can only alter M the velocity and direction, but never affect the area deſcribed, it is plain that any number of ſucceſſive impulſes will likewiſe have no effect in altering the area. Suppoſe the number of impulſes to be infinite, or, in other words, let a force directed to the center act continually on the body, and a polygon with an infinite number of ſides, that is to ſay, a curve, will be deſcribed, whoſe radius accompanying the moving body, will deſcribe or ſweep over equal areas in equal times.

And converſely, if a body revolve about a point, N ſo as to deſcribe a curve, whoſe radius accompanying the body, ſhall ſweep over equal areas in equal times, the centripetal force which deflects the motion from a right line, muſt be directed to that point.

But no inſtance of a centripetal force directed to O an immoveable point is found in nature. Bodies attract one another, and that mutually. Therefore,

fore, if one body revolves about another, this laſt will not remain at reſt, bur will revolve in a * ſimilar curve about the common center of gravity, as will alſo the firſt body. That is to ſay, if the center of gravity be at reſt, the two bodies will abſolutely move in ſimilar curves about that center, and relatively about each other in curves ſimilar to thoſe laſt mentioned.

P These motions will not be altered, if the center of gravity be ſuppoſed in motion (78, T. 79, W).

Q Therefore, when we ſpeak of the orbits and periodical revolutions of bodies, we may in general regard one of the bodies as ſtationary, and the other as revolving round it.

R If a body revolve round a center in an orbit which is not circular, it is plain, that to deſcribe equal areas in equal times, it muſt move ſwifter when near the center than when more diſtant; and it is likewiſe evident, that when the velocity, and conſequently the tendency to fly off in a tangent is increaſed, a greater centripetal force will be required to retain it in its orbit.

S From the properties of the ellipſis it is demonſtrated, that a body revolving in that curve, whoſe centripetal force tends to one of its foci, muſt in any part of its orbit be attracted towards that focus, by a force which is reciprocally as the ſquare of its diſtance †.

---

* Principia, I. 57.

† Principia, I. 11.

A very complete and clear idea of the ellipſis T may be had from the common way of deſcribing it; if a thread CAC (fig. 41.) be faſtened by its ends at the points C C, and a pointed inſtrument be inſerted in the bight or bend at A, and moved towards B or E, keeping the thread at full ſtretch, it will in one revolution deſcribe the ellipſis ABDE. C C are called the foci.

To illuſtrate this doctrine of revolving bodies, U we may obſerve, that as gravity conſtantly acts on all bodies in the vicinity of the earth, attracting them towards its center, every projectile, which is not thrown in the line of the perpendicular, may be conſidered as a body revolving about that center; and if its orbit be not ſufficiently large to contain or circumſcribe the body of the earth, it will be interrupted in its courſe, and remain at reſt ſomewhere on the ſurface. Thus let ABE (fig. 42.) repreſent the earth, whoſe center is at C; then if a body be projected from A in the direction AF, it will, by the action of the centripetal force, be deflected into the curve AGB, and will remain at reſt at B, being prevented from deſcribing the whole orbit AGBDA, by the body of the earth, which interrupts its courſe at B. But the part AGB of the elliptical orbit of a projectile is ſo ſmall, in compariſon to that part which is not deſcribed, that it may without any ſenſible error be conſidered as a parabola, except ſo far as the reſiſtance of the air, which is not here regarded, makes it fall ſhort of B, by deſtroying part of its motion.

v The orbit AGBDA, of which the parabola is part, would have been deſcribed upon the ſuppoſition, that the attraction towards the center continues to obſerve the ſame law within as without the ſphere. But this ſuppoſition, however, is not true; for a ſphere of uniform denſity, compoſed of particles which attract each other with forces reciprocally as the ſquares of their diſtances, will attract bodies without its ſurface according to the ſame law; relation being had to its center. But the centripetal forces of bodies placed within the ſphere, will be directly as their diſtances from the center *.

w Let the circle BCDG (fig. 43.) repreſent the earth. From the top of the mountain A, let a body be projected in the horizontal direction AF, with a force that will carry it to B on the ſurface. Imagine it to be projected in the ſame direction with a ſtill greater force, and it will be carried to C. A ſtill greater increaſe of force will carry it to D. And a yet greater augmentation will carry it round the earth to A, where it will proceed with a velocity equal to that with which it was firſt projected, and by conſequence, the reſiſtance of the air being diſregarded, will revolve in that orbit for ever. But if the projectile force be ſtill more increaſed, it will deſcribe the ellipſis A b A with an unequable motion; ſlower at b and ſwifter at A, and continue to revolve for ever in that orbit.

---

* Principia, I. 80.

UNIV. OF MICH.

If gravity acts in the diſtant ſpaces of the heavens inverſely according to the ſquares of the diſtances, it will be eaſy to apply this to the motions of the celeſtial bodies. We ſhall again reſume this ſubject; but in the mean time it is neceſſary, that the appearances ſhould firſt be deſcribed before an explanation of them can be given.

BOOK I.

SECT. III.

Aſtronomy.

CHAP. I.

CONCERNING THE SYSTEM OF THE UNIVERSE.

THERE are two methods by which knowledge may be acquired and announced; namely, by analyſis, or by ſyntheſis. In the method of analyſis the proceſs is made from things that are compounded to things that are more ſimple. Cauſes, or firſt principles, are inveſtigated by attending to, and examining their effects. But in the method of ſyntheſis the proceſs is directly the contrary; for here the cauſes, or firſt principles, being known or aſſumed, are made by compoſition or combination to account for their effects. It is very manifeſt, that in the acquiſition of philoſophical knowledge the former method muſt be firſt made uſe of. We ſee no ſimple events in nature, and we cannot come at cauſes but by analyſing the effects we behold. Thus it is that firſt principles are obtained, which may afterwards be extended by ſyntheſis, to account for other phenomena in a more univerſal manner. Generally ſpeaking, the analyſis appears beſt adapted for acquiring knowledge, while the ſynthetical method is more convenient and conciſe for communicating it, when known.

2 In the communication of the knowledge that relates to the heavenly bodies, and is termed aſtronomy, we might aſſume as eſtabliſhed firſt principles every thing which reſpects their mutual poſitions and motions abſolutely conſidered, and from thence deduce ſynthetically the phenomena that would appear to a ſpectator placed on the earth or elſewhere. Or we might ſtill more generally, from the laws of motion aſſumed as firſt principles, deduce the conſequences that would ariſe from the motions of bodies in circumſtances ſuch as the heavenly bodies are known to be placed in. By theſe methods the ſcience would be moſt expeditiouſly taught, and even ſuppoſing the firſt principles to be merely aſſumed at hazard, or gratuitouſly, yet their conſtant agreement with the events that happen in nature might furniſh no inconſiderable preſumption of their truth. However, it is clear, that the knowledge on which the ſynthetical reaſoning is built, muſt be either aſſumed gratuitouſly, or obtained by analyſis. It is not eaſy, nay, it is perhaps impoſſible, to form any conception how a finite intelligence can, to any advantage,

make uſe of firſt principles taken without a previous uſe of the analytical method of deduction. Yet the whole hiſtory of natural philoſophy affords numberleſs inſtances of men of real abilities who have indulged their vanity and indolence in deducing conſequences from principles entirely hypothetical, and often falſe. Far from wiſhing to imitate theſe, we ſhall not aſſume, even eſtabliſhed truths, without giving their proofs where they can be explained with that facility that our intention demands. Inſtead, therefore, of deducing the apparent phenomena from the real motions of the heavenly bodies, it is preſumed that it will be much more intereſting, though rather more prolix, to note the obvious appearances, and thence infer their cauſes. Theſe inferences have been the conſequence of the obſervation and ſtudy of ſeveral of the moſt diſtinguiſhed men of genius in the courſe of many ages. The truth has been acquired by ſlow degrees, and indirect methods. It has often been obſcured by the admixture of error. Its progreſs has been retarded by the operation of prejudice, and the pride of falſe ſcience has with difficulty given way to its force. When diſcoveries are completed, it is eaſy to trace the moſt direct ſteps by which they might have been made, though it has ſcarcely ever happened that the diſcoverers proceeded by thoſe ſteps. Let us therefore paſs over in ſilence the various and intricate ſchemes made uſe of to ſolve the celeſtial appearances before the ancient ſyſtem of the world was revived by Copernicus [*], and ſince eſtabliſhed for ever by the immortal Newton. Let us imagine ourſelves in the open air buſied in the contemplation of the phenomena that occur in the heavens, and while we note the facts, and make plain deductions from them, we ſhall be inſenſibly led to the knowledge of that beautiful regularity and order that prevail through the immenſe regions of ſpace, and evince the intelligence and power of the incomprehenſible Firſt Cauſe.

The firſt and moſt obvious phenomenon that A preſents itſelf to obſervation, is the apparent diurnal motion of the heavens, by which the ſun, moon, and ſtars are ſeen to riſe and ſet. This motion is obſerved to be ſubject to ſeeming irregularities. If its period be eſtimated from ſun-riſe to ſun-riſe, a little time evinces, that the ſun does not always riſe at the ſame point, nor remain above the horizon ſo long in winter as in ſummer. The moon is ſtill leſs adapted to the purpoſe of determining this period, its variations being in every reſpect more conſpicuous. The ſtars remain, which appear indeed to riſe and ſet regularly, but yet in a period ſhorter than the natural day; for thoſe ſtars, which at a certain time of the year are ſeen to riſe at midnight, are found to make their appearance early in the evening, after the ſpace of three months is elapſed. It is therefore to be determined, which of thoſe motions ought to be regarded as the motion of the heavens; and it is much more obvious and intelligible, to ſuppoſe, that the ſun, by a relative motion to the eaſtward with reſpect to the fixed ſtars, ſhould make the days ſomewhat longer than the real time of a revolution, than that all the ſtars, while they preſerve their mutual diſtances unaltered, ſhould conſtantly move with a velocity greater than that of the heavens. To determine this relative path of the ſun is not difficult. By the ſhadow of a perpendicular ſtaff or other equivalent inſtrument at mid-day, its varying declination towards the north or ſouth may be known, and the advance in the riſing of the ſtars will mark its difference in right aſcenſion. By this, or ſome ſuch method, it may be diſcovered, that while the fixed ſtars riſe and ſet, each on its proper points of bearing or poſition, without varying their relative ſituations; the ſun, by deſcribing annually a circle towards the eaſt, inclined to the direction of its daily courſe in an angle of 23½ * degrees, muſt occaſion all the difference of ſeaſons, length of days, &c.

---

[*] A.D. 1543, the year of his death, After ſuppreſſing his book, "de Revolutionibus orbium celeſtium," for more than thirty-ſix years, it was at length publiſhed, and a copy brought him a few hours before his death. Gaſſendus in vita Copernici. See alſo Sir John Pringle's elegant "Diſcourſe on the Attraction of Mountains."

---

* Every circle is ſuppoſed to be divided into 360 parts, which are called degrees.

B In noting theſe appearances, it is natural to ſelect the brighteſt ſtars as objects of our attention. The planetary bodies will on this account be the early objects of our notice. The planet Venus eſpecially, receding from the ſun to the eaſtward, will appear as an evening ſtar in the weſt after ſunſet; and afterwards will diſappear on its re-approach to it, and be ſeen at a nearly equal diſtance to the weſtward, and riſing before the ſun, become a morning ſtar. The ſlowneſs of its apparent motion near its greateſt elongation or angular diſtance from the ſun, evinces that it is moved in an orbit, near the center of which the ſun is placed; and the ſhort time employed in paſſing from its greateſt elongation eaſtward to its greateſt elongation weſtward, when compared with the time of its courſe between the ſame elongations in the contrary direction, ſhews that its revolution is made from weſt to eaſt. The proportion between its diſtance from the ſun, and that of the ſun from the earth, may be found from the quantity of its greateſt elongation.

C To illuſtrate this, let S (fig. 44.) repreſent the ſun, E the earth, IMKN the orbit of Venus, CSD part of the ecliptic, or ſun's apparent annual path in the heavens. Then to a ſpectator at E, ſituated nearly in the plane of the planet's orbit, the planet when at B will be referred to the point b in the ecliptic; and when by its abſolute motion in its orbit, it has deſcribed the arc BU, it will appear to have deſcribed the arc bD. When at U, it will appear ſtationary at D, and after a little time begin to move back from D to b: for after deſcribing the arc U J, it will again be ſeen at b. Continuing its courſe, it will arrive at V, having apparently paſſed through the arc b c. At V it will again become ſtationary and afterwards move to G, F, A, &c. which will be repreſented by their correſponding points in the ecliptic. Now, ſince the motion, as ſeen from the earth, is that which appears in the ecliptic, and ſince the apparent motion from b to D may as well be produced by a real motion from J to U as by one from B to U, it remains to be determined in what direction the real motion is made by which the apparent motion is produced. Now, becauſe E C and E D are tangents to the orbit, the points D and C, which correſpond with the poſitions U and V, are thoſe of its greateſt elongations; and becauſe the arc U V, which is paſſed over in the inferior part of the orbit between the two greateſt elongations, is leſs than the ſuperior arc V A B U, which is paſſed over between the ſame elongations, it is plain, that when the planet is in the inferior part of the orbit, the ſpace C D will be performed in leſs time than when it is in the ſuperior part. It is alſo evident, when the planet moves in the ſuperior part of its orbit, that the apparent motion in the ecliptic has the ſame direction as the real motion. Therefore, ſince we have a criterion to diſtinguiſh the motion in the ſuperior from that in the inferior part, we can eaſily determine the direction of the motion in its orbit, which is proved to be from weſt to eaſt.

The diſtance of Venus from the Sun, in proportion of that of the Sun from the Earth, is determined from its greateſt elongation: thus draw the line V S, which will be at right angles to the tangent V E; then in the right angled triangle V E S, by the rules of plain trigonometry,

As radius
Is to the ſine of the angle of greateſt elongation V E S,
So is the Sun's diſtance from the earth E S,
To the diſtance of Venus from the Sun V S.

By ſimilar obſervations on the planet Mercury, it is determined that its revolutions are performed round the ſun in the ſame manner, becauſe they are accompanied with circumſtances of the ſame nature as appear in the motion of Venus.

CHAP. II.

OF THE FIGURE AND MOTION OF THE EARTH.

WE have not yet conſidered the effects which the ſun's annual motion in the ecliptic has upon the apparent motions of the planets, though it is very conſiderable: neither have we determined whether this apparent annual motion be the conſequence of a real motion of the ſun about the earth, or of the earth round the ſun. The cœleſtial phenomena may be explained either way, but in a much more ſimple and intelligible manner by the latter ſuppoſition. We ſhall therefore previouſly give an account of the figure of the earth, and the reaſons on which the ſuppoſition of its motion is founded. The proof will come more properly when we treat of the phyſical cauſes of theſe motions. At preſent we only deſcribe appearances, and draw plain inferences from them.

G The purpoſes of aſtronomy require, that the fixed ſtars ſhould be claſſed into conſtellations. When their relative ſituations are known, it muſt ſoon be perceived, that their diurnal revolutions are performed round an axis, obliquely ſituated with reſpect to the horizon, or circle that bounds our view; one of its extremities or poles being above the horizon to the * north, and the other as far below it to the

---

* In this elucidation the obſerver is ſuppoſed to be in north latitude.

ſouth, and conſequently inviſible. By travelling to the northward, the north pole is obſerved to become more elevated, and that exactly in proportion to the ſpace travelled over; from which circumſtance it follows, that the earth is round or ſpherical.

For, let I B E, fig. 46. repreſent a plane ſection of the earth at right angles to its ſurface; let A B repreſent the plumb line or perpendicular, and B P the line of direction, in which the * pole ſtar is ſeen; the angle A B P will then be the complement of the altitude of the ſtar: let the ſame ſtar be ſeen from another point E in the line of direction E Q, which, by reaſon of the great diſtance of the ſtar, may be eſteemed parallel to B P; the angle D E Q will then be equal to the angle A B P, or complement of the former altitude at B. From E draw the plumb line or perpendicular E F; the angle D E F will then be the difference between the two co-altitudes A B P, F E Q, or between the two altitudes; but this difference is equal to the angle B C E, formed between the plumb lines, and is proportional to the arc or diſtance B E †.

Now,

---

* There is no ſtar ſituated at the pole. The ſtar α in Urſa Minor, which is called the pole ſtar, is about 2 ½ degrees diſtant from it.

† Let the part H N of the line D F (fig. 46.) be comprehended between the perpendiculars A H, F N, which, if continued, meet at G. Conceive H N to be divided into an indefinite number of equal parts, by means of perpendiculars B I, O K, &c. ſeverally prolonged till they meet. Then in the triangle H G I, the ſides, H G, I G, are equal, becauſe oppoſed to equal angles at H and I: and for a like reaſon, in the triangle

Now, if the angle formed between two perpendiculars to a given line be always in proportion to that part of the line comprehended between the perpendiculars, the line itſelf is circular: and that ſolid, whoſe ſections, paſſing through two oppoſite points, are all circles, is itſelf a ſphere.

I Hence, if the length of the arc B E be meaſured, and its quantity in degrees known by obſervation on a ſtar, the length of the whole circumference of the earth may be found by this proportion. As the quantity of degrees is to the length meaſured, ſo is the whole circumference, or 360 degrees to its length.

K The modern circumnavigation likewiſe proves the ſphericity of the earth; for, by ſailing continually eaſtward, or continually weſtward, veſſels arrive again at the port from whence their firſt departure was taken.

L Alſo, in an eclipſe of the moon, the ſhadow of the earth is always projected in a circular form. Now, it is evident, that the body, whoſe ſhadow is in all poſitions a circle, muſt itſelf be a globe.

M Unfurniſhed with thoſe proofs, which the ſagacity and more accurate obſervations of later ages

angle I G K, the ſides I G, G K are equal. The ſame proceſs of argumentation will extend to prove, that all the ſmall triangles between H and N are iſoſceles, having a common vertex at G; that is to ſay, there is no definite part of H N from which a right line can be drawn to the point G, either greater or leſs than H G: H N is therefore a curve, having ſuch a relation to a certain point, that all right lines drawn from it to that point are equal; or it is circular. Which was to be ſhewn.

ages have afforded, the ancients could not adduce thoſe reaſons for the earth's motion, that depend on the general laws of motion, and the nature of gravity. Without doubt they had recourſe to thoſe, which depend on the moral fitneſs of things. They were perſuaded that the wiſdom of the Creator had formed every thing in the beſt manner poſſible, and therefore, that when an effect could be as well produced by ſimple as by complicated cauſes, the obſerver of nature ought to attribute it to the former. They ſaw the two planets, Mercury and Venus, revolving round the ſun in orbits, whoſe radii are leſs than the diſtance between the ſun and the earth: the ſuperior planets, Mars, Jupiter, and Saturn, were alſo obſerved to move in orbits about the ſun, but at greater diſtances than that between the ſun and earth. If the ſun were ſuppoſed to move abſolutely in the ecliptic or its apparent path, it muſt carry the orbits of theſe bodies along with it, and conſequently their abſolute motions muſt be very complicated; but if the earth be ſuppoſed to deſcribe an orbit round the ſun, between Venus and Mars, the abſolute motions become ſimple and natural, and an admirable uniformity prevails throughout the ſyſtem.

The annual motion of the earth being allowed on this principle, its diurnal motion would follow by the ſame argument; it being much more reaſonable and conſiſtent to ſuppoſe, that the earth, by a daily revolution on its own axis from weſt to eaſt, ſhould occaſion the apparent motion of the cœleſtial bodies, N than that thoſe bodies ſhould, beſides their other various motions, have that aſtoniſhing velocity which a real diurnal motion would produce. The objections common obſervers might make would be eaſily diſproved by men whoſe penetration was capable of going thus far. From the obſervations by which the ſpherical form of the earth was diſcovered, they would alſo gather, that bodies fell not abſolutely down, or in a direction referable to pure ſpace, as was imagined, but always in a line directed towards the center of the earth, and conſequently that no danger of bodies falling off would ariſe from its continual change of poſition. The inſtances of ſhips carried by the tides in calm weather would likewiſe ſerve to ſhew, that the relative motions or poſitions of bodies are not changed by an equal velocity given to them in the ſame parallel direction.

CHAP.

CHAP. III.

OF THE MUTUAL APPEARANCES OF THE SUPERIOR AND INFERIOR PLANETS.

O THAT the planets Mars, Jupiter, and Saturn revolve in orbits, which include the orbit of the earth, is evident, becauſe they are frequently ſeen in the part of the ecliptic directly oppoſite to the ſun; and that the orbits reſpect the ſun as a center, appears as well from thoſe oppoſitions which happen in every part of the ecliptic, as from their unequable apparent motions, which are explained by referring them to that center.

P We have conſidered the apparent motions of the inferior planets as far as relates to their ſituation with reſpect to the ſun. The motion of the earth affects thoſe appearances, to ſpeak in general, only by retarding the time they employ to return again to the ſame ſituation.

Q The earth at E (fig. 44.) is a ſuperior planet with reſpect to Venus. A ſpectator on Venus at B would ſee the earth E elongated from the ſun under the angle E B S; which angle of elongation would increaſe by the motion of Venus in its orbit from B to U, where it becomes a right angle E U S. From J it would be ſeen in an angle of ſtill greater elongation E J S, and from M it would be ſeen directly in oppoſition to the ſun. Paſſing from M to K, V, &c.

the angle of elongation would decreaſe till the arrival of Venus at N, whence the earth would be in conjunction with the ſun, and the angle of elongation would vaniſh. This relative motion of the ſuperior planet with reſpect to the ſun is contrary to the order of the ſigns, or from eaſt to weſt, and depends entirely upon the motion of the inferior planet on which the ſpectator is ſuppoſed to be placed.

R If the earth E was at a diſtance indefinitely great, the lines B E, U E, J E, &c. might be eſteemed parallel, and conſequently the ſpectator would behold it always in the ſame point of the ecliptic, its ſituation with regard to the ſun being varied only by the apparent motion of the ſun, occaſioned by the real motion of Venus. But as this is by no means the caſe, an apparent motion of the earth among the ſigns of the ecliptic will be produced. Thus, the earth viewed from N, will appear among the fixed ſtars at P; from B it will appear at R; from U at O, where it will be ſtationary ſo long as the orbit of Venus does not ſenſibly differ from its tangent; from J it will be ſeen returned back to R with a retrograde motion; from M at P; from K at T; from V at Q, where it again becomes ſtationary; and from A it will be again ſeen at T, its motion having again become direct: whence we may obſerve, that

S When a ſuperior planet viewed from an inferior appears ſtationary, the inferior planet viewed at the ſame time from the ſuperior is alſo ſtationary; and,

T When the inferior planet viewed from the ſuperior moves apparently retrograde, or contrary to the order of the ſigns, the ſuperior planet has alſo an apparently retrograde motion.

U But ſince the earth has an annual motion round the ſun in its orbit, (110, M) we are therefore to diſcover what part of the apparent motion of Venus is produced by that cauſe. It is plain, that if the earth were at reſt, and Venus ſeen at U, its greateſt elongation, it would again be ſeen in the ſame poſition, after performing a complete revolution in its orbit. But while Venus is performing this revolution, the earth is carried from E towards W, and ſo forth. Therefore Venus muſt paſs between two ſimilar elongations, not only a complete revolution, but likewiſe the whole angular ſpace which the earth has performed in the ſame time. Hence its periodical time may be found. For the time between two ſimilar poſitions is obſerved to be 583 days. Now, dividing the earth's orbit into 365 equal parts or days, the angular velocity of Venus will be denoted by the angular ſpace paſſed over in the given time, namely, one revolution, or 365 days added to 583 days, equal to 948; and the earth's angular velocity will be 583.

V The periodical times of Venus and the Earth will be reciprocally as their angular velocities; conſequently,

As the angular velocity of Venus -	948
Is to the angular velocity of the earth	583
So is the periodical time of the earth	365
To the periodical time of Venus	224½

Were it not for the fixed ſtars, it would be impoſſible to diſcover or obſerve the annual motion of the earth. We ſhould conclude, that each planet made a complete revolution between any two ſimilar ſituations with reſpect to the ſun, becauſe the ſpaces of elongation are ſimilarly deſcribed, and are in quantity the ſame, whether the earth be in motion or not. Thus, if the earth be fixed at E, the ſame apparent elongations will be made by Venus with any velocity whatſoever in its orbit, but they will occur more frequently the greater the velocity. If a motion be given to the earth in the orbit EW, Venus will approach from U to M, which is now in motion, with a velocity equal to the difference between its angular velocity and that of the earth: or if the earth's angular velocity be greateſt, it will apparently recede from M, and deſcribe its revolutions in the contrary direction to its real motion. Now, as all the apparent motion of Venus in elongation is known by its approach or receſs from the line SE, and ſince any angular motion of SE can only change the relative velocity of Venus; and ſince a change of velocity will not alter the elongations, except as to time, it is evident, that we cannot determine whether E be at reſt or no, from the appearances of the planets which revolve about the ſun. It is then from the apparent motion of the ſun, with reſpect to the fixed ſtars, that we conclude that the earth deſcribes an orbit in about 365 days.

If the ſuperior planet E be at reſt, the retrograde X
motion of the inferior planet U among the fixed ſtars will be the ſame as its motion in elongation, viz. the angle UEV. But if E move in the ſame direction as U, but angularly ſlower, the arc deſcribed by the retrograde motion in the ecliptic will be leſs than that deſcribed between the two oppoſite elongations. The ſame is true of the retrograde motion of the ſuperior viewed from the inferior planet.

For the motion of E towards W cauſes an ap- Y
parent motion of the ſun towards D. And as the retrograde motion of U referred to the arc DS is ſloweſt near the elongations, it is plain that U will not become ſtationary in the ecliptic till its apparent motion in elongation from D towards S is equal to the ſun's apparent motion in the contrary direction; that is to ſay, till ſome time after paſſing the greateſt elongation, ſuppoſe at d. After which its motion muſt become retrograde till it arrives at H, equidiſtant from its greateſt elongation on the other ſide, where it will again become ſtationary, its apparent motion in elongation being equal and contrary to that of the ſun in the ecliptic. Now, the angle HEd is leſs than the angle of retrograde motion in elongation CED. And ſince the angle IEq is equal to HEd, it is alſo leſs than CED. But thoſe angles

IEq and HEd are the meaſures of the retrograde motions of the ſuperior and inferior planets, when viewed from each other. Whence the propoſition is evident.

CHAP. IV.

OF THE SUPERIOR PLANETS, AND OF THE TRUE FORM OF THE PLANETARY ORBITS.

Z THE appearance of the earth when viewed from Venus being explained, it will be eaſy to apply that explanation to the apparent motions of the ſuperior planets. Of the two inferior planets Venus ſerved us as an inſtance; and of the three ſuperior ones we ſhall ſelect Jupiter, as being the moſt bright and conſpicuous. The motions of this planet being accounted for, ſimilar obſervations and ſimilar reaſoning will obviouſly ſolve thoſe of the other planets, whoſe particular phenomena will not, therefore, require a more minute elucidation.

A That the planet Jupiter revolves in an orbit, which includes that of the earth, and reſpects the ſun as its center, was ſhewn in the beginning of the laſt chapter; and its apparent motions are obſerved to be ſimilar to thoſe which it was proved the earth would have when ſeen from Venus. It remains to diſcover its periodical time and diſtance from the ſun.

B Let S (fig. 47.) repreſent the ſun, E the earth, J Jupiter, the circle Eea the earth's orbit, and the circle J j A the orbit of Jupiter. Suppoſe Jupiter to be in oppoſition to the ſun. The earth revolving in its orbit will, in the ſpace of 365 days, arrive again at E, but the oppoſition will not then happen, becauſe Jupiter in the mean time will have moved in its orbit towards j. The earth muſt therefore paſs through the arc E e or 33 ½ days before it overtakes it. Conſequently, the angular velocity of Jupiter will be denoted by 33 ½, and that of the earth by one whole revolution, (or 365) added to 33 ½, equal to 398 ½. But as the periodical times are reciprocally as the angular velocities, it will be

* As the angular velocity of Jupiter 33 ½
Is to the angular velocity of the earth 398 ½
So is the periodical time of the earth 365 days
To the periodical time of Jupiter 4340 days.

The periodical time of Jupiter being thus obtained, it will be eaſy to determine its † heliocentric place at any time before or after the oppoſition, and the proportion of its diſtance from the ſun to that of the earth from the ſun being known, its ‡ geocentric place may likewiſe, at any time, be diſcovered. Its proportional diſtance is thus found.

The figure as before. Suppoſe the earth to have moved from E to c, in a given time. From the time may be found the quantity of the angle E S C; and in the ſame time Jupiter will have moved to B, the angle J S B being alſo known from its proportion to his whole periodical revolution. Subtract the angle J S B from the angle J S C, and the remainder will be the angle B S C. By obſervation find the angle B C S, or Jupiter's elongation, from the ſun. In the triangle C B S, the ſum of the two angles B S C and B C S being taken from 180 degrees, leaves the angle C B S. Then, by plain trigonometry,

---

* Smaller fractions being rejected, the periodical times are not here exact.

† Viewed from the ſun as a center.

‡ Viewed from the earth as a center.

E As the ſine of the angle of the earth's elongation, when viewed from Jupiter	C B S
Is to the ſine of the angle of Jupiter's elongation, when viewed from the earth	B C S,
So is the earth's diſtance from the ſun	C S
To Jupiter's diſtance from the ſun	B S.

F The angle of the earth's elongation, when viewed from Jupiter, is called Jupiter's annual parallax, and is always equal to the difference between its heliocentric and geocentric place in the ecliptic, as a little conſideration will ſhew.

G By ſimilar obſervations on the other ſuperior planets, it is found that their apparent motions are attended with circumſtances of the ſame nature as thoſe of Jupiter. The ſame conſequences muſt therefore follow reſpecting their orbits, periods, and other affections.

H Thus far we have ſpoken of the appearances of the planets, as if their revolutions were performed in circular orbits, in the center of which the ſun was ſuppoſed to be placed. But this is not the caſe. Conjunctions, oppoſitions, ſimilar elongations, or other mutual ſituations of the planets, do not return again in exactly the ſame time, and their diſtances from the ſun are found to be greater or leſs in different parts of their orbits, their angular velocities being always greater when the diſtances are leſs. Thus, by the increaſed diameter of the I ſun during the winter half-year, we find that the earth's diſtance is diminiſhed; and that its velocity is increaſed, is evinced from the apparent motion of the ſun, by which it paſſes through the winter half-circle of the ecliptic in near eight days leſs than it employs to deſcribe the ſummer-half. By a variety of obſervations of elongation or parallax, the relative or proportional diſtances of the planets from the ſun, and their velocities are found for every heliocentric poſition. Whence K they are proved to revolve in elliptical orbits, the ſun being placed in one of the foci; and their velocities are ſuch, that a radius drawn from the ſun to the planet, and ſuppoſed to move with it, deſcribes equal areas in equal times.

The diſtance between the center s (fig. 41.) L and one of the foci c of an elliptical orbit, is called its eccentricity. The two extreme points of the tranſverſe or longeſt diameter, L and U, are called the apſides. If the focus about which the equal areas are deſcribed be at c, the point L neareſt that focus is called the lower apſis, and U

is

is called the upper apſis; the diameter UL being called the line of the apſides. But it is more common to ſay, that a planet is in its perihelium when at L, and in its aphelium when at U. When the earth is in its perihelium, the ſun is ſaid to be in its perigee, and when the earth is in its aphelium, the ſun is ſaid to be in its apogee.

M From the generation of the ellipſis (97, T) it is evident, that the whole length of the ſtring is equal to the tranſverſe diameter: becauſe, when the point A is at U, the return of the ſtring from U to the neareſt focus C, is exactly equal to the part between L and the other focus C, where there is no ſtring. Let GH be the conjugate, or ſhorteſt diameter of the ellipſis, and CG will be equal to half the ſtring, or half the tranſverſe diameter. Suppoſe now a planet to revolve in the ellipſis about the focus C neareſt L. Then CL will be its neareſt diſtance, CU its greateſt diſtance, and CG its mean diſtance. For, CG equal to SL, exceeds the leaſt diſtance CL by the quantity CS, which is juſt as much as it falls ſhort of CU, the greateſt diſtance. If therefore the mean diſtance CG, and the eccentricity CS of a planet be given, its orbit may be deſcribed: becauſe the greateſt diſtance is equal to the ſum of the eccentricity and the mean diſtance; and the leaſt diſtance is equal to their difference.

N The eccentricities of the planets are ſo ſmall, that their orbits approach nearly to circles, If the plane of the earth's orbit were extended indefinitely every way, it would mark that circle in the heavens which is called the ecliptic, or ſun's path. If the orbit of any other planet be ſituated in this plane, it will always be ſeen in the ecliptic, whether viewed from the earth or the ſun. But if the plane of the planet's orbit be obliquely ſituated with reſpect to that of the ecliptic, it will interſect it in a line paſſing through the center of the ſun, and the planet will never be ſeen in the ecliptic but when in the points of interſection. Theſe oppoſite points of the ecliptic are called the nodes, and the line of interſection is called the line of the nodes. When a planet croſſes the ecliptic from ſouth to north, the node is termed the aſcending node; and when it croſſes from north to the ſouthward, the node is termed the deſcending node.

The orbits of all the planets are inclined to the ecliptic in ſmall angles.

CHAP. V.

OF THE AFFECTIONS OF THE PLANETS.

Q BY ſuppoſing ourſelves in the place of one of the ancients who diſcovered the order of the planetary ſyſtem, we have diſplayed in a curſory manner ſome of the moſt obvious phenomena, and pointed out their natural conſequences. What has been ſaid is ſufficient to ſhew to thoſe who are totally unacquainted with the ſubject, that the doctrine of a ſyſtem of bodies revolving round the ſun is not merely ideal, but founded on the moſt natural deduction from the celeſtial appearances. For the proceſſes by which the planets places are determined in elliptical orbits, we refer the reader to treatiſes written expreſsly on the ſubject, and in the mean time proceed to note ſeveral of thoſe affections of the heavenly bodies, as determined by the accurate obſervations of modern times.

R Seven planets, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgium Sidus *, revolve about the ſun in orbits included within each other, in the order here uſed in mentioning their names, Mercury being neareſt the

---

* This planet was diſcovered in the year 1781, by William Herſchel, Eſq. a native of Hanover.

Sun. Theſe are called primary planets, beſides which, there are ten which are called ſecondary planets, Moons or Satellites. The ſecondary planets reſpect the primary planets, performing their revolutions about them, but are at the ſame time carried round the Sun in the orbit of the primary. Saturn is attended by five Moons, Jupiter by four, and the Earth by one, all which, except the laſt, are inviſible to us, by reaſon of their ſmallneſs and diſtance, unleſs teleſcopes be made uſe of. Without this inſtrument, it would likewiſe be impoſſible to aſcertain the apparent diameters of any of the cœleſtial bodies, the Sun and Moon excepted. The following table exhibits ſome of the affections of the primary planets.

Anno 1784.	Mercury.	Venus.	Earth.
Greateſt poſſible elongation of inferior and parallax of ſuperior planets, -	28°. 20′	47°. 48′	* *
Proportional mean diſtances from the ſun, - -	38710	72333	100000
Periodical revolutions,	87 d. 23 h. 15½ m.	224 d. 16 h. 49¼ m.	365 d. 6 h. 9¼ m.
Diurnal rotations, - -	Unknown.	23 h. 22 m.	23 h. 56 m. 04 ſ.
Inclinations of their orbits to the ecliptic, - -	7°. 00′	3°. 23⅓′	* * *
Eccentricities, - -	7960	510	1680
Place of the aſcending node, - -	1 ſ. 15 deg. 46½ m.	2 ſ. 14 d. 44 m.	* * *
Place of the aphelium,	8 ſ. 14 d. 13 m.	10 ſ. 9 d. 38 m.	9 ſ. 9 d. 15¼ m.
Proportion of axis to the equatorial diameter, - -	Unknown.	Unknown.	2289 to 2300
Greateſt apparent diameters, - -	11″	58″	*
Diameters, if ſeen at the ſun's mean diſtance, or proportional diameters,	7″	16″. 7	17″. 3.
Diameters in geographical miles; that of the ſun being 762490, -	2782	6637	6875½
Mean diſtances from the ſun in ſemidiameters of the earth,	9210	17210	23799.
Mean diſtances from the ſun in geographical miles, -	31671900	59181700	81818400
Proportions of light, -	668	191	100

Mars.	Jupiter.	Saturn.	Georgium Sidus, 1782.
47°. 24′	11°. 51′	6°. 29′	3°. 4¼′
152369	520098	953937	1903421
686 d. 23 h. 30¾ m.	4332 d. 8 h. 51½ m.	10761 d. 14 h. 36¾ m.	30445 d. 18 h.
24 h. 39 m. 22 ſ.	9 h. 56 m.	Unknown.	Unknown.
1°. 51′.	1°. 19⅓′	2°. 30⅓′	46′. 12″
14218	25277	53163	4759
1 ſ. 17 d. 59 m.	3 ſ. 8 d. 50 m.	3 ſ. 21 d. 48½ m.	3 ſ. 13 d. 1 m.
5 ſ. 2 d. 6¼ m.	6 ſ. 10 d. 57½ m.	9 ſ. 0 d. 45½ m.	11 ſ. 23 d. 23 m.
1272 to 1355	13 to 14	Unknown.	Unknown.
25″	46″	20″	4″
9″ 1	193″. 7	171″. 7	76″. 2
3617	76982	68238	30310
36262	123778	227028	453000
124666000	425536000	780395100	1557350000
43	3. 7	1. 1	0. 276

CHAP. VI.

OF PARALLAXES, AND OF THE TRANSIT OF VENUS.

S ONE of the moſt uſual methods of meaſuring inacceſſible diſtances, is by means of two ſtations whoſe diſtance from each other is known, and the angles formed at each ſtation between lines ſuppoſed to be drawn from the diſtant object to them, and the line that joins the ſtations to each other. Thus the diſtance between A and B (fig. 48.) being known, as likewiſe the angles CAB and CBA, the diſtance AC or BC may be readily found by plane trigonometry.

T Suppoſe the object C, when viewed from B, (fig. 48.) to coincide with another object S, which is at a diſtance indefinitely great; then the object C will not appear to coincide with S, when viewed from A. For S, on account of its great diſtance, will be ſeen in the line AS, parallel to BS; and C will be ſeen in the line AC, the angle SAC being the difference between the apparent places from A and B. This angle, becauſe of the parallels AS and BS, will be always equal to the angle ACB, and is by aſtronomers called the parallax. It is uſually diſtinguiſhed by ſome appellation relative to the nature of the line AB: for inſtance, it is called the annual parallax, when AB is the radius of the annual orbit; the diurnal parallax, when AB is the ſemidiameter of the earth, &c.

UNIV. OF MICH.

It is almoſt unneceſſary to obſerve, that the longer u
the line A B is in proportion to the diſtance of C, the greater the angle A C B, and that in moſt practical caſes, the greater the angle the leſs is the diſtance affected by any ſmall error. It is therefore requiſite that the baſe A B be as large as poſſible or convenient.

The diſtances of the planets were found by tri- v
gonometry, the diſtance of the earth from the ſun being aſſumed as a baſe. But as that baſe cannot actually be meaſured, the ſaid diſtances are only proportional or relative, the baſe being ſuppoſed to be divided into 100000 equal parts; but whether thoſe parts are miles, leagues, or anſwer to any other denomination of length, was not determined. The real diſtances muſt be diſcovered by a parallax whoſe baſe is known.

The diameter of the earth is the longeſt right line, w
whoſe length we can obtain from admeaſurement, and is in general the baſe uſed for determining the diſtances of celeſtial objects by their parallax, which parallax is found as follows:

Let A O B (fig. 49.) repreſent the earth, C its cen- x
ter, and Z the zenith or point in the heavens, ſituated perpendicularly over the point O at its ſurface. Then C H will be the rational horizon, and O K the ſenſible horizon. Suppoſe a ſpectator at C views a celeſtial object at Z, the revolution of the earth will cauſe it to move apparently through the quadrant Z H in ſix hours, at the end of which time he will ſee it in the horizon at H. But to a ſpectator at O it will ap-

pear in the horizon when at K, after paſſing through the apparent quadrant or right angle Z O K, in a time as much leſs than ſix hours as the arc Z K is leſs than Z H, or 90 degrees. Hence the time of an object's paſſing between the zenith and ſenſible horizon being known, the angle O K C, or horizontal parallax, may be found. For, as ſix hours is to 90 degrees, ſo is the time obſerved, to the arc Z K, which being taken from 90 degrees, leaves the arc K H meaſuring the angle K C H, which is equal to O K C, or the horizontal parallax.

Y The horizontal parallax being diſcovered, the diſtance of the object follows by this analogy; in the triangle O K C.

As the horizontal parallax, ſine - - O K C
Is to the earth's ſemidiameter - - - O C
So is radius - - - ſine - 90°
To the diſtance - - - - C K

Z The fixed ſtars have no parallax, either horizontal or even annual, whence it follows, that their diſtances are beyond all compariſon greater than that of the earth from the ſun.

A It is obviouſly unneceſſary in obſervations of parallax to wait till the object has deſcribed the whole apparent quadrant Z K (fig. 49.): for, when it is arrived at E, the angle Z C E may be known from the time, and Z O E from obſervation, and their difference will be the angle of parallax O E C: ſo that in the triangle O E C are given two angles, and the ſide O C, from whence the other parts are eaſily found.

Obſervations of parallax are conveniently made B
by the help of the fixed ſtars. Thus, if the object
E when at Z be ſeen in a given poſition with reſpect
to a fixed ſtar, it will continue to have the ſame
poſition when arrived at E, provided the ſpectator
be at C; but if the ſpectator be at O it will be ſeen
depreſſed below its former poſition by the quantity
of the parallax D O E, becauſe the ſtar has no parallax,
but is ſeen in the ſame apparent place either from O
or C.

The Sun's parallax is ſo exceedingly ſmall, that C
the beſt inſtruments in the hands of the moſt ſkilful
obſervers, have ſcarcely effected more than to ſhew
that it has one. To remedy this, the horizontal D
parallaxes of the nearer planets have been attempted,
particularly of Mars, when in oppoſition to the Sun,
this planet being then as near again to the Earth
as the Sun is, and has therefore a parallax twice as
great. But as this parallax is not found to exceed
half a minute of a degree, the unavoidable uncertainty
of obſervation, and other cauſes, render it
not ſufficiently exact to determine the Sun's diſtance
within a 30th part of the whole. It is eaſy to comprehend
how the Sun's diſtance may be found when
the diſtance of Mars, from the Earth, in oppoſition,
is known. Thus, if S (fig. 50.) be the Sun, E
the Earth, and M Mars, in oppoſition, then E M
will be the diſtance of Mars from the Earth, and
alſo the difference between M S and E S, or the reſpective
diſtances of Mars and the Earth from the

Sun. The proportional diſtances are known. Therefore,

As the difference between the proportional diſtances of Mars and the Earth from the Sun,

Is to the proportional diſtance of the Earth from the Sun;

So is the diſtance between the Earth and Mars in oppoſition, or the difference between their real diſtances from the Sun,

To the Earth's real diſtance from the Sun.

a, Several other methods were deviſed by the ancients for diſcovering the Sun's parallax, which, though they ſhew the ſagacity and penetration of their inventors, are leſs ſufficient for the purpoſe than the foregoing. We ſhall therefore omit mentioning them, and give a ſhort explanation of that for which we are indebted to the great Dr. Halley, by which the diſtance of the Sun is determined with greater accuracy than by any other method.

f The planet Venus, as has been ſhewn, paſſes the Sun twice in revolving from any poſition of elongation to the ſame poſition again (105, c). At thoſe times this planet is ſaid to be in conjunction with the Sun.

e When the planet Venus is ſituated in a line between the Sun and the Earth, it is ſaid to be in its inferior conjunction; and when it is in the oppoſite part of its orbit, the Sun being in a line between it and the Earth, it is ſaid to be in its ſuperior conjunction. If the orbits of the Earth and Venus were in the ſame plane, it is evident that Venus would paſs behind the Sun with a direct motion every ſuperior conjunction, and would paſs over its * diſc, or before it, with a retrograde motion every inferior conjunction. But as Venus's orbit is inclined to the ecliptic in an angle of about 3½ degrees, this planet will, in general, paſs to the northward or ſouthward of the ſun, and will only be viſible on its diſc when the inferior conjunction happens at or near one of the nodes. This happens but once (or ſometimes twice at an interval of about 8 years) in more than 120 years.

To ſhew how this tranſit is applied to the purpoſe of finding the Sun's diſtance, we ſhall paſs over thoſe elements that enter into the computation previous or ſubſequent to actual obſervation, and ſhall only explain the general principles on which the method is founded. H

Let S (fig. 51.) repreſent the Sun, E the Earth, V, U, W, the planet Venus in different poſitions, the arc L N a part of the Earth's orbit, and the arc O M a part of the orbit of Venus. Then, becauſe the angular velocities of Venus and the Earth are known, as alſo their proportional diſtances, it will be eaſy to compute the time Venus will employ in paſſing through the arc V W, which, when viewed from the Earth, is equal to the known diameter (or chord) of the Sun C D; the heliocentric value or length of the arc V W may likewiſe be readily found. Suppoſe then an obſerver at A on the I

---

* Surface.

Earth's ſurface to view the planet Venus at v, it will appear juſt entered within the Sun's diſc at c, and paſſing in the arc v w, will appear to deſcribe the line c D, arriving at D at the end of the computed time. But during this time the obſerver will, by the Earth's diurnal revolution, be carried from A towards P; and arriving at P at the ſame inſtant that Venus arrives at u, will behold the tranſit juſt finiſhing at D; conſequently it will be of a duration proportionally as much ſhorter than the computed time, as the heliocentric arc v u is ſhorter than v w. The arc v w is known by computation, therefore, ſince Venus's motion may in very ſmall arcs be reckoned uniform,

As the computed time
Is to the computed arc v w,
So is the obſerved time
To the arc - v u;

which being taken from v w, leaves the arc u w, that ſubtends the angle u D v. This laſt angle is the parallax of the baſe A P; and the baſe A P is found by the analogy

As one day or 24 hours
Is to the circumference of the earth (or parallel of latitude)
So is the obſerved time
To the arc A P, whoſe chord is the baſe,

K But becauſe the minuteſt errors in a buſineſs of this nature are of very great conſequence, and becauſe the length of the arc v w, depending on the Sun's diameter, can ſcarcely be obtained by calculation to that extreme degree of exactneſs, which is requiſite, it is adviſable to take another obſervation on a place ſo ſituated on the Earth, that the obſerver being carried in a direction apparently contrary to the former, the errors may counteract each other.

Let the repreſentations be as in the laſt figure. If the ſun have declination at the time of the tranſit, B (fig. 52.) will repreſent the pole towards which the Sun declines. The obſerver at A, if at reſt, would behold the tranſit during the time Venus paſſes from v to w, but being by the Earth's diurnal revolution carried from A through the arc A E P to P, and arriving at P at the inſtant in which Venus arrives at U, he will perceive the tranſit juſt finiſhing at D; conſequently its duration will be as much longer than the computed time as the heliocentric arc v u is longer than v w. v u being found by the before mentioned analogy, the difference between v u and v w is w u, or the parallax of A P, as before. L

Now, in theſe two caſes, a ſimilar error will have a contrary effect in the firſt to that which it has in the latter. For if by any error, the computed arc v w (fig. 51.) be taken too large, the arc U W, and conſequently the parallax will come out too great. But in the latter obſervation, if the computed arc v w (fig. 52.) be taken too large, the arc w u, and conſequently the parallax will come out too little. Therefore the mean between M two ſuch obſervations will be much more to be depended on than either ſingly.

By obſervations on the tranſits of Venus over the Sun in the years 1761 and 1769, the Sun's mean parallax was found to be 8$\frac{2}{3}$ ſeconds, and hence the Sun's diſtance is deduced to be very near 11900 diameters of the Earth, or 81818400 * geographical miles.

The laſt three articles in Chap. V. concerning the affections of the planets are deduced from this diſtance; for,

As the proportional diſtance of the earth
Is to its real diſtance,
So is the proportional diſtance of any other planet
To its real diſtance.

---

* A geographical mile is $\frac{1}{60}$ part of a degree of the earth.

CHAP. VII.

OF THE SECONDARY PLANETS.

THE ſecondary planets, as was before obſerved, are ten in number, five of which deſcribe orbits about the planet Saturn, four about Jupiter, and one accompanies the Earth. The ſecondaries of Saturn and Jupiter are obſerved by the teleſcope, and by their motions in elongation to the eaſtward or weſtward of their primaries is obtained the knowledge of their diſtances and periodical times, in the ſame manner as has been already ſhewn and explained in the planet Venus (105, c). Saturn is likewiſe attended by a phenomenon, which to us appears to be a large broad ring, of no viſible thickneſs. Its breadth is equal to its diſtance from the body of the planet, and its diameter is to that of Saturn as 9 to 4. The moſt probable conjecture is, that it conſiſts of a vaſt number of ſatellites, which revolve in, and enlighten that region. o

Of Saturn's five moons, the periodical times: First, $1^{d}\ 21^{h}\ 19^{m}$; Second, $2^{d}\ 17^{h}\ 41^{m}$; Third, $4^{d}\ 13^{h}\ 47^{m}$; Fourth, $15^{d}\ 22^{h}\ 41^{m}$; Fifth, 79^{d}, 22^{h}, 41^{m}.—Diſtances in ſemidiameters of the ring, Firſt, $1\frac{95}{100}$; Second, $2\frac{5}{10}$; Third, $3\frac{52}{100}$; Fourth, $8\frac{9}{100}$; Fifth, $23\frac{71}{100}$.—Of Jupiter's four moons, the periodical times: Firſt, $1^{d}\ 18^{h}\ 27^{m}\ 33^{s}$; Second, $3^{d}\ 13^{h}\ 13^{m}\ 42^{s}$; Third, p q

Third, $7^{d}\,3^{h}\,42^{m}\,33^{s}$; Fourth, $16^{d}\,16^{h}\,32^{m}\,8^{s}$. Diſtances in ſemidiameters of Jupiter: Firſt, $5\frac{667}{1000}$; Second, $9\frac{17}{1000}$; Third, $14\frac{384}{1000}$; Fourth, $25\frac{299}{1000}$.

All the planets, both primary and ſecondary, receive their light from the Sun. This is evident, becauſe that face only is enlightened which is turned towards that luminary, as may be more particularly ſeen in our Moon, a greater or leſs part of which is viſible, according to the poſition in which we lie for viewing the illuminated face. The ſame varieties are ſeen in the planets Mars and Venus, not to mention the tranſits of Venus and Mercury over the Sun, at which time they appear as black unenlightened ſpots. The phaſes of Jupiter and Saturn are always round and full, becauſe the Earth is ſo near the Sun in reſpect to their diſtances, that their dark ſide can never be ſenſibly turned towards us; yet, that they are opake, is evident from the diſappearing of Jupiter's moons when they enter into its ſhadow. And though by reaſon of their vaſt diſtance the like obſcurations of the ſatellites of Saturn cannot be obſerved, yet we can plainly ſee that the ring caſts a ſhadow on its body: whence we may be certain of the opacity of both: for if the ring were not opake it could caſt no ſhadow, and if Saturn ſhone by any native light of his own, the interception of the Sun's light would cauſe no defect or ſhadow on his body. It is unneceſſary to obſerve, that the Earth and its Moon are illuminated only on that part or ſide on which the Sun ſhines.

s When one planet intercepts any part of the Sun's light from another, the planet from which the light is intercepted is ſaid to be eclipſed, if it be a ſecondary. But if they are both primaries, the inferior planet is ſaid to make a tranſit. When the moon intercepts the Sun's light from the Earth, it is uſual to ſay the Sun is eclipſed, though, properly ſpeaking, it is the Earth that is eclipſed.

T There are three ways in which the ſatellites of Jupiter or Saturn may diſappear from an obſerver placed on the Earth. Thus, let S (fig. 53.) repreſent the Sun, E the Earth in its orbit, J the planet Jupiter and its moons. Then the outermoſt ſatellite, for example, will diſappear on the enlightened face of Jupiter when at its inferior conjunction M. It will alſo diſappear at its ſuperior conjunction N, being hid behind the body of the planet. And laſtly, it will diſappear when at O, being eclipſed in paſſing through the ſhadow of Jupiter.

u From theſe conſiderations is obtained a good method of finding the parallax of the Earth's annual orbit. For which purpoſe the inſtant of the ſatellite's firſt diſappearance behind the body of Jupiter muſt be carefully obſerved, as likewiſe the inſtant of its re-appearance: the middle inſtant will be that of the ſuperior conjunction at N. In like manner, the middle-inſtant of the eclipſe at O muſt be found. The time the ſatellite employs in paſſing through the arc N O will thus be known, and conſequently the angle N J O. For,

As the periodical time of the ſatellite
Is to the time of paſſing the arc N O,
So is the whole orbit or 360 degrees
To the angle N J O.

But the angle N J O is equal to the angle E J S, or the annual parallax.

V By the obſervations of theſe eclipſes, the diſcovery of the longitude on ſhore is eaſily obtained, but the violent motion of ſhips at ſea has hitherto prevented the uſe of teleſcopes on board proper for this purpoſe, though there are good reaſons to believe that this W difficulty is not inſurmountable. From theſe obſervations it alſo appears, that light is not propagated from luminous bodies in an inſtant, but paſſes through a given ſpace with an aſſignable velocity. This velocity is extremely great, for it paſſes through the whole diſtance between the Sun and the Earth in about eight minutes; that is to ſay, at the rate of one hundred and ſeventy thouſand miles in a ſecond of time. For the periodical times of the ſatellites being known, it is not difficult to determine the preciſe time of any of their eclipſes. But it is found neceſſary to make an allowance for the poſition of the Earth with reſpect to Jupiter, ſince the eclipſes happen ſooner when the Earth is at F, (fig. 53.) in its orbit, than when at a greater diſtance, ſuppoſe at E; and as it is abſurd to ſuppoſe, that the poſition of the Earth ſhould ſenſibly and equally affect the periodical revolutions of bodies ſo vaſtly remote, and revolving in ſuch different periods, it is an opinion univerſally received, and

UNIV. OF MICH.

and confirmed by other obſervations on the light of the fixed ſtars *, that the eclipſes happen later when the Earth is at E than when at F, becauſe the light muſt in the latter caſe paſs through a ſpace as much greater as the line JE exceeds JF.

CHAP. VIII.

OF THE MOON.

THAT the Moon revolves round the Earth, is proved from its apparent diameter, which continues at all times, and in all poſitions, nearly of the ſame magnitude, whence it may be eaſily inferred, that its diſtance from the Earth is nearly at all times the ſame. Its horizontal parallax, which at a medium is about 57′, ſhews that it is very much nearer to us than the reſt of the celeſtial bodies.

The moſt remarkable appearance in the Moon is the continual change of figure to which it is ſubject. Sometimes it appears perfectly full or circular, at other times half illuminated, and at other times more or leſs than half; changing through a very great variety of figures. Theſe changes being always the ſame at the ſame elongation from the Sun, are a proof that it receives its light from that luminary: for the Moon is enlightened only on the ſide that faces the Sun; and a greater or leſs quantity of that enlightened part is viſible to us, according to our

---

* The aberration of the fixed ſtars ariſing from the progreſſive motion of light, will be explained in Book II. Sect. I.

poſition. This cannot be better illuſtrated than by an ivory ball, which being held in the Sun in various poſitions, will preſent a greater or leſs part of its illuminated ſide to the view of the obſerver. If it be held nearly in oppoſition, ſo that the eye of the obſerver may be almoſt immediately between it and the Sun, the greateſt part of the enlightened ſide will be ſeen. But if it be moved in a circular orbit towards the Sun, the viſible enlightened part will gradually decreaſe, and at laſt diſappear when the ball is held directly towards the Sun. Or, to apply the experiment more immediately to our preſent purpoſe; if the ball at any time, when the Sun and Moon are both viſible, be held directly between the eye of the obſerver and the Moon, that part of the ball on which the Sun ſhines will appear exactly of the ſame figure as the Moon itſelf.

z The Moon's path or orbit is inclined to the plane of the ecliptic, in an angle of about five degrees and
A a quarter. Its periodical revolution is performed in twenty-ſeven days, ſeven hours, forty-three minutes, eleven ſeconds and a half; but becauſe, during that time the Sun, by its apparent motion, advances conſiderably in the ecliptic, a ſpace of about two days and a quarter is required by the
B Moon to overtake it. When the Moon is as nearly in a line between the Earth and the Sun as the inclination of its orbit will allow, it is called the New
C Moon; and when the Earth is in like manner between the Moon and the Sun, the Moon is ſaid to be

be full. The time between two ſucceeding full D
moons is called the ſynodical revolution, and
exceeds the periodical revolution, for the reaſon
already given, it being performed in twenty-nine
days, twelve hours, forty-four minutes, and three
ſeconds. If the new or full Moon happen near the E
node, an eclipſe takes place; at the new Moon,
the Moon being interpoſed between the Sun and
Earth, occaſions an eclipſe of the Sun; at the full,
the Moon entering into the ſhadow of the Earth, is
deprived of the Sun's light, the Earth being interpoſed
between it and the Sun: which phenomenon
is called a lunar eclipſe, or eclipſe of the Moon.
At other times, that is, when the new or full Moon
happens at a diſtance from the node, the Moon paſſes
too far to the northward or ſouthward of the ecliptic,
either to intercept the Sun's light from the
Earth, or to enter the Earth's ſhadow, and conſequently
no eclipſe happens.

F It is determined from obſervations of angular velocity, parallax and apparent diameter, that the Moon revolves round the Earth in an elliptical orbit, in the focus of which the Earth is placed: and that its velocity is ſuch, that a radius joining its center with that of the Earth does very nearly deſcribe equal areas in equal times.

G The line of the apſides, or principal diameter of the Moon's orbit, is not fixed or ſtationary, but revolves with an irregular or libratory motion from weſt to eaſt: completing one revolution in almoſt nine years.

H The line of the nodes is alſo ſubject to a like irregular motion from eaſt to weſt, which is completed in almoſt nineteen years.

I The variation of the Moon's motion in any part of its orbit is the difference between its real motion and that which it would have had, provided it had deſcribed equal areas in equal times. This is governed chiefly by its elongation from the Sun. During the firſt quarter its velocity is diminiſhed; in the ſecond quarter, from the quadrature to the oppoſition or full Moon, it is increaſed; in the third quarter, from the oppoſition to the laſt quadrature, the velocity is again diminiſhed; and from that quadrature to the conjunction, its velocity is again increaſed. The quantity of angular motion loſt exceeds the quantity gained: therefore the whole periodical revolution is performed in a longer time than would have been employed if the Moon were ſubject to no ſuch variation, but deſcribed equal areas in equal times.

K This variation, and conſequently the retardation of the periodical time, is greater when the Earth is in the perihelium, and leſs when the Earth is in the aphelium: whence it comes to paſs, that all the Moon's revolutions are not equal, but are performed in leſs time in the latter ſituation than in the former.

L On all theſe, as well as other accounts, the determination of the Moon's place in the heavens for a given inſtant of time has ever been a problem of great difficulty, which till of late years has not been ſolved to any conſiderable degree of exactneſs. Within the laſt twenty years the commiſſioners, appointed by the Engliſh government for the diſcovery of the longitude, have particularly attended to this branch of aſtronomy, and by publiſhing almanacs in which the Moon's elongation from the Sun, and from certain fixed ſtars, is aſcertained for every three hours, have enabled navigators to determine the ſituation of ſhips at ſea in general within thirty miles of the truth. This is an advantage of ſingular uſe in long voyages, and is at preſent much uſed in the royal navy, and Eaſt India Company's ſhips.

CHAP. IX.

CONCERNING THE ECLIPSES OF THE SUN AND MOON.

WE have ſeen in what manner the periodical revolutions of the celeſtial bodies, together with the figure, magnitude, and poſition of their orbits, may be reſpectively determined by obſervations made on their apparent motions and ſituations. From the properties of the ellipſis, and the eſtabliſhed law of their velocities (121, K), or otherwiſe, more immediately from the conſideration of gravity (98, W, X), aſtronomical tables are computed, by which the places of the heavenly bodies may be found for any inſtant of time. The

conſtruction and uſe of theſe would lead us too far from the conciſe and exterior view of phenomena that our limits require: we ſhall therefore aſſume,
M as a thing granted of courſe, that the place of any celeſtial body may be found for any given inſtant of time.

N The eclipſes of the Sun and Moon are phenomena that command the attention even of the vulgar, who have always retained a ſuperſtitious veneration for the ſcience of aſtronomy, chiefly on account of the means it affords of foretelling events of this nature. And though in reality the knowledge required in calculating an eclipſe does not eſſentially differ from that employed in determining the time of the riſing and ſetting of the Sun or Moon, yet, there is no doubt but a more particular attention to this ſubject will be acceptable to the reader.

O As the ſhadows of the Moon and Earth are the cauſes of eclipſes, it will be neceſſary firſt to determine their figure. Becauſe the Sun, the Earth, and the Moon, are ſpherical bodies, it follows that the ſhadows of the two latter muſt be either conical
P or cylindrical; that is to ſay (fig. 54.), if the Sun I K be leſs than the Earth C D, the ſhadow of the latter will be part of a cone, whoſe ſection is terminated by the lines C E, D F, and whoſe baſe is indefinitely diſtant: or, if the Sun A B be equal to the Earth C D, the ſhadow will be a cylinder between the lines C G, D H, whoſe baſe is indefinitely diſtant. In either caſe the ſhadows of the Earth

Earth may conſequently fall upon and eclipſe the
ſuperior planets, when in direct oppoſition to the
Sun. But this never happens, and therefore the Sun
is neither leſs than, nor equal to, the Earth, but
greater. We know moreover, from the Sun's parallax
(136, N), that it is much greater than the
Earth, becauſe the Sun's diameter ſeen from the
Earth is about 32 minutes, whereas the Earth's Q
diameter ſeen from the Sun is (126.) only about
17 ſeconds, a quantity that may be regarded as
inſenſible, or inconſiderable in many obſervations.
And ſince the Sun exceeds the Earth in ſo high a R
proportion, it muſt of neceſſity be yet greater with S
regard to the Moon, becauſe this laſt is leſs than
the Earth. Let AB (fig. 55.) repreſent the Sun
greater than the Earth CD. The rays of light AC, BD,
paſſing from the extreme edges of the Sun, and in
contact with the Earth on the ſame ſide, will afterwards
meet or croſs in the point K. No part of
the Sun's light will appear within the cone CKD,
which is therefore the ſhadow in which an obſerver,
being placed, would be totally deprived of the Sun.
But there will be a partial ſhadow or penumbra
between thoſe rays ADM, BCL, that paſs from the
extreme edges of the Sun, and touch the oppoſite
extremes of the Earth: that is to ſay, an obſerver
between the lines CL and DM, but without the dark
cone CKD, will ſee only a part of the Sun, the reſt
being hidden by the interpoſition of the Earth: the
quantity of the Sun thus obſcured will be greater,
and the penumbra darker, the nearer the obſerver is

placed to the cone C K D. Laſtly, if the obſerver be ſituated beyond the vertex of the dark ſhadow K, between the lines K N, K O, formed by the continuation of the extreme rays, he will behold the exterior parts of the Sun forming a lucid ring, environing the Earth on all ſides.

T The angle C K D, at the vertex of the Earth's ſhadow, is equal to the difference between the diameter of the Sun, ſeen from the Earth or angle A C B, and the diameter of the Earth ſeen from the Sun, or angle C B D*. Or, if the Earth's apparent diameter from the Sun (147, Q) be rejected as inconſiderable, the angle of the ſhadow will be equal to the Sun's apparent diameter.

U The angle C I D, at the vertex of the penumbra, is equal to the ſum of the diameter of the Sun ſeen from the Earth, or angle A C B, and the diameter of the Earth ſeen from the Sun or angle C A D: or, if the Earth's apparent diameter from the Sun (147, Q) be rejected as inconſiderable, the angle of the penumbra is equal to the Sun's apparent diameter.

V The apparent diameter of any ſection E F, of the ſhadow, ſuppoſed to be viewed from the Earth, namely, the angle E D F, is equal to the exceſs of the Earth's apparent diameter ſeen from the place of ſection, namely, the angle C E D, beyond the angle at the vertex of the ſhadow C K D: or, if the angle of the ſhadow (148, T) be taken as equal to the Sun's apparent diameter, the apparent diameter of any ſection of the ſhadow ſeen from the Earth will be equal to the difference between the apparent diameters of the Sun and Earth, as ſeen from the place of ſection, this laſt diameter being greateſt.

---

* Euclid I. 32. is repeatedly uſed in what immediately follows: that is to ſay, in any triangle B C K, the outward angle, A C B formed by prolonging one of its ſides, is equal to the ſum of the two inward oppoſite angles C K D, C B D: and conſequently, that one of the two laſt-named angles, or C K D, will be equal to the difference between the external angle A C B and the other interior oppoſite angle C B D.

The apparent diameter of any ſection G H, of the penumbra, ſuppoſed to be ſeen from the Earth, namely, the angle G D H, is equal to the ſum of the Earth's apparent diameter ſeen from the place of ſection, namely, the angle C G D, added to the angle at the vertex of the penumbra C I D. Or, if the angle of the penumbra (148, U) be taken as equal to the Sun's apparent diameter, the apparent diameter of any ſection of the penumbra ſeen from the Earth will be equal to the ſum of the apparent diameters of the Sun and Earth, as ſeen from the place of ſection. W

Every thing that has been here ſhewn reſpecting the ſhadows of the Earth is true in like circumſtances of the Moon (147, S). X

To apply theſe obſervations to the facts, let A B (fig. 56.) repreſent the Sun, C D the Earth, and I K or L the Moon in its orbit K M N; let the Moon be at I K, between the Sun and Earth; its total ſhadow may then entirely deprive a part of the Earth at O of the Sun's light, and its penumbra will cauſe a partial eclipſe of the Sun to the inhabi- Y z tants between G and H. Again, ſuppoſe the Moon to be at L, and it will itſelf be eclipſed by the interpoſition of the Earth between it and the Sun. In lunar eclipſes, the Earth's penumbra is not attended to, becauſe its effects in obſcuring the Moon cannot be obſerved with preciſion by a ſpectator placed on the Earth.

A It has already been obſerved (143, E), that eclipſes can only happen when the Moon is near one of the nodes of its orbit. Let A B M (fig. 57.) repreſent the Sun, viewed from the Earth, C D a portion of the ecliptic, or Sun's apparent path, and E F a part of the orbit of the Moon; which planet is repreſented at different times by the circles G, H, I. It is evident, that the eclipſe or obſcuration of the Sun entirely depends on the poſition of the node N, and the angle of inclination F N D. If the angle of inclination remain unaltered while the node N is very remote from the center K of the Sun, the points K and L may be farther apart than to permit any occultation or apparent contact: and it is clear, that an enlargement of the angle F N D may produce the ſame effect: on the contrary, an approach or coincidence of N with K, or a diminution of the angle F N D may cauſe an eclipſe, the quantity of obſcuration in which will be ſo much greater, as theſe circumſtances are more prevalent.

B The Sun's place K in the ecliptic (146, M) being known from tables, together with the inclination of the Moon's orbit, the place of the node, and of the Moon itſelf, as likewiſe the apparent diameters

of the luminaries reſpectively, it will be eaſy to
find the velocity of the Moon in elongation, and
conſequently the beginning, middle, end, quantity
of obſcuration, and other requiſites concerning the
eclipſe. If the computation be made from the ta- C
bular places of the heavenly bodies, the reſult will
give the eclipſe as ſeen from the center of the Earth,
becauſe, in all tables where the Earth is ſpoken of,
that center is meant, except otherwiſe mentioned.
But it is required to determine the particulars of the D
eclipſe for a given place on the Earth's ſurface, and
this includes the conſideration of parallax. The
Sun's parallax being very minute (136, N) may in
this, and moſt other caſes, be rejected: but the
Moon's parallax is ſo great, that it is at leaſt of as
much conſequence as any other element whatſo-
ever. For, on this account, the Moon's apparent
path as ſeen from the ſurface of the Earth, is ſo
different from that which it would have when beheld
from the center, that the ſame conjunction which
gives a total eclipſe at one place ſhall not occaſion
the ſmalleſt obſcuration of the Sun when beheld at
the ſame inſtant from another part of the Earth.

This method of computing a ſolar eclipſe is very E
operoſe. For the Moon's parallax at any time paſt
or to come cannot be had without finding its al-
titude by ſpherical trigonometry, and other com-
putations muſt then be made to deduce the ap-
parent poſitions of the Sun and Moon with their
relative velocity, and ſo forth. And becauſe the
altitude of the Moon is continually changing, it is

neceſſary to repeat the computations of parallax from time to time. To render this buſineſs leſs tedious, it has been found expedient to conſider the phenomena of ſolar eclipſes as they would appear to an obſerver placed on the Moon. Let AB (fig. 58.) repreſent the Earth, ſeen from the Moon, under an angle of about 1°. 54′, or double the Moon's horizontal parallax (141, X), the line CD a portion of the oppoſite part of the Moon's orbit in which the Earth is ſeen, the circles G, H, I, ſhadows of the Moon, that on account of their always being diametrically oppoſite the Sun, will be found to ſenſe in the ecliptic, eſpecially when the Moon is near the node. The path FE of the ſhadow will therefore make an angle FND with the line CD, equal to the inclination of the Moon's orbit, and the interſection N will be as far diſtant from the center of the Earth as the node is heliocentrically from the center of the Moon. Now the motion of the Earth, in the line CD, is equivalent to the Moon's apparent motion in its orbit ſeen from the Earth, and the motion of the ſhadow is equivalent to the Sun's motion in the ecliptic. Conſequently, the center of the Earth and ſhadow of the Moon may be projected as ſeen from the Moon. The diameter of the dark ſhadow K, L, or M, ſeen from the Moon, will be equal to the exceſs of the Moon's apparent diameter beyond that of the Sun, when both are ſeen from the Earth (148, V. 149, X), the Moon's apparent diameter being greateſt, but if it be the leſs of the two, the ſhadow will not reach the Earth. The diameter of the penumbra G, H, or I, ſeen from the Moon, will be equal to the ſum of the apparent diameters of the Sun and Moon, ſeen from the Earth (149, W, X). With theſe data the eclipſe may be conſtructed univerſally.

But in conſtructing the eclipſe for a particular place, the rotation of the Earth on its axis muſt be brought into conſideration. For while the ſhadow paſſes over the Earth's diſc, a given place P will be carried round in its parallel of latitude, and may likewiſe be marked in the projection for any inſtant of time. When the place enters the penumbra, the eclipſe will begin there; when the place and the center of the ſhadow are the neareſt, the obſcuration will be greateſt, and when the penumbra leaves the place, the eclipſe will end. If there be a dark ſhadow, and it paſſes over the place, the eclipſe will be total. If there be no dark ſhadow, and the place ſhould paſs within the penumbra to a depth exceeding the Moon's apparent diameter, the Sun will be ſeen environing the Moon on all ſides; whence the eclipſe is ſaid to be annular (148, S). And, in general, in any ſolar eclipſe that is not annular, the diſtance of the place within the penumbra will meaſure the greateſt ſection or part of a diameter of the Sun obſcured at that inſtant, and the line joining the cuſps, or angular termination of the apparent part of the Sun, will be at right angles to the meaſuring line or diameter, O meter, of which the meaſuring line repreſents a part.

H We are now to conſider an eclipſe of the Moon. It is evident, that the difference in the phenomena of a ſolar eclipſe would not take place if the parallax of each luminary were the ſame; becauſe, whatever mutation of place the parallax might occaſion in the one, the ſame would be produced in the other, and they would neither approach nor recede from each other on that account. Now the ſection of the Earth's ſhadow paſſed through by the Moon in a lunar eclipſe, being at the ſame diſtance from the Earth as the Moon itſelf, muſt be ſubject to the ſame parallax at equal altitudes; and ſince the individual points of immerſion, emerſion, or other periods of the eclipſe muſt in the ſhadow have the ſame altitudes as the parts of the Moon they, as it were, lie on and obſcure, the effects of parallax muſt be the ſame on both. Rejecting therefore the conſideration of parallax, the Earth's ſhadow A B (fig. 59.) may be taken to occupy a place in the heavens diametrically oppoſite the Sun, and having an equal and ſimilar motion to the apparent motion of that luminary, its apparent diameter, ſeen from the Earth, will be equal to the difference between the apparent diameters of the Earth and Sun, as ſeen from the Moon (148, v). Or it will be equal to twice the horizontal parallax of the Moon diminiſhed by the ſubtraction of the Sun's apparent diameter. And if the inclination of the orbit of the Moon be found, there will be a certain diſtance of the node N from the center of the ſhadow C, that will require the Moon near the oppoſition to paſs through the Earth's ſhadow, and be conſequently eclipſed. From the greater or leſs diſtance of the node N, or M, it will be determined whether the eclipſe will be partial or total; and from the reſpective places, the quantity and direction of the relative velocity, together with the apparent magnitudes of the ſhadow and the Moon, all the particulars of the eclipſe may be known without difficulty.

It may with great reaſon be demanded, how it I happens that the Moon, which is affirmed to emit no light of itſelf, but only by reflection of the Sun, is nevertheleſs ſufficiently luminous, even in the very middle of a total eclipſe, to be diſtinctly ſeen of a duſky reddiſh color. The Earth's atmoſphere, or body of air that ſurrounds it, is the cauſe of this phenomenon. In fact, the ſhadow of the K Earth itſelf never extends ſo far as the Moon's orbit, though the ſhadow occaſioned by the diſperſion or reflection of the light that falls on the atmoſphere may, with a very ſmall allowance, be taken for the ſhadow which the Earth would have had if the light had paſſed cloſe by it without interruption. We cannot with regularity explain the refraction of light in this place. It will therefore be ſufficient to obſerve, that in the event now under conſideration the Sun's light falling obliquely on the atmoſphere, is bent or turned out of its courſe ſo as to converge ſooner to a point, than it would

otherwiſe have done; the ſpherical atmoſphere per-
forming, in ſome meaſure, the office of a large
convex lens, or burning-glaſs. The more obliquely
the rays fall, the greater is their deviation from
their original courſe; and thoſe rays that paſs cloſe
to the Earth are found, by obſervations on the
ſetting Sun and other heavenly bodies, to ſuffer a
refraction of about 33 minutes of meaſure. The
laws of optics, hereafter to be explained, require
that they ſhould undergo an equal refraction in
paſſing out through the oppoſite part of the at-
L moſphere. Each exterior ray of the real ſhadow
will therefore paſs 66 minutes within the rays
that would have formed the cone CKD, fig. 55.
and conſequently, the angle at the vertex of the
cone will be 132 minutes, or 2°. 12′ greater than
it would have been; that is, it will be equal to
the diameter of the Sun 32 (148, T), added to
M 2°. 12′, which gives 2°. 44′. Hence the axis of
the cone, or length of the ſhadow, is found to be
no more than 42 ſemidiameters of the Earth; where-
as the radius of the Moon's orbit, or mean di-
ſtance of the Moon, is about 60 ſemidiameters
N of the Earth. In the ſpace between the penumbra
and the Earth's real ſhadow it is much darker than
the penumbra, though that ſpace is illuminated by
the rays of the Sun, which are variouſly refracted, ac-
cording to the denſity of the air they paſs through.
Many rays are reflected back, and the rays that go
forward are ſuch whoſe nature does not admit of
their being eaſily reflected. We are to ſhew in

future that theſe are the red, orange, and yellow. Hence it is that the Moon in an eclipſe appears red; and a ſpectator on the Moon would, after loſing ſight of the Sun, behold the Earth envivironed with a narrow luminous edge of bright red light, ſhaded off with yellow on the outſide.

Since the Earth, when beheld from the Moon, muſt always appear in the part of the heavens immediately oppoſite the Moon's apparent place as ſeen from the Earth, the enlightened ſide of the Earth will have the ſame figure, when ſeen from the Moon, as the dark ſide of the Moon would exhibit if it could be ſeen at the ſame inſtant from the Earth. Thus, when the Moon is inviſible, or near the conjunction, the Earth is in oppoſition, and preſents a full luminous face to the Moon; and on the contrary, when the Moon is at the full, or oppoſite the Sun, it muſt be on the dark ſide of the Earth, which conſequently then becomes inviſible. Near the beginning of the firſt or end of the laſt quarter, the dark ſide of the Moon is rendered viſible by the full Earth ſhining on it, but is ſcarcely ſo luminous as the Moon when eclipſed. The Earth's diſc, ſeen from the Moon, is about thirteen times that of the Moon ſeen from the Earth. If the Earth reflected as great a part of the light that falls on it as the Moon does, its light at the Moon would exceed the moon-light with us in that ratio. This, however, is not probable, though it may fairly be ſuppoſed that it is three or four times as great. But we have already obſerved that the Earth's atmoſphere, in a lunar eclipſe, illuminates the Moon rather more than this. Whence it follows, that the narrow ring of light encircling the Earth, when ſeen from the Moon during an eclipſe, gives a light far exceeding our moon-light at the full; or even that of the Earth's full face ſhining on the Moon: and as the ſurface of the ring, by computation, can hardly equal the one hundredth part of the Earth's diſk, or the one eighth part of the Moon's diſk, its brightneſs will be more than twenty-four times that of the Moon. It muſt conſequently be very luminous and dazzling.

Notwithſtanding this, there have been eclipſes of the Moon, when in that part of it's orbit near the Earth, in which that luminary entirely diſappeared. But theſe obſervations are very rare.

propa
Planeta
Fig.

UNIV. OF MICH.

CHAP. X.

OF COMETS; AND OF THE PROPORTION OF LIGHT AND HEAT ON THE PLANETS.

BESIDES the ſeven primary planets already R
enumerated, and their moons or attendants,
there are other bodies that revolve round the Sun,
and claim peculiar diſtinction on ſeveral ac-
counts. Theſe are called Comets, and appear oc-
caſionally in every part of the heavens; their
motions being performed in very long ellipſes,
whoſe lower focus is in or near the Sun. By ob- S
ſervations of parallax it is found, that at their
firſt appearance they are nearer to us than Jupiter;
whence it is concluded, that they are moſt com-
monly leſs than that planet; for if they were as
large as Saturn, they would be ſeen as far off.

When a comet arrives within a certain diſtance T
of the Sun, it emits a fume or vapour, which is
called its tail. This ſhews that they contain a por-
tion of matter conſiderably more rare and volatile
than any on the Earth; for the tail begins to ap-
pear while they are yet in a higher, and conſe-
quently colder region than Mars. The tail is al-
ways directed to that part of the heavens which is
directly or nearly oppoſite to the Sun; and is
greater after the comet has paſt its perihelium, than
during its approach towards it; being greateſt of

all at the time when it has juſt paſt the perihelium.

That part of a comet's orbit which comes under our obſervation is ſo ſmall in proportion to the whole, that in moſt it does not differ from a parabola, by quantities that obſervation can diſtinguiſh: for which reaſon the dimenſions of their orbits and periodical times cannot be determined with any degree of preciſion from a ſingle appearance. But from the re-appearance of comets after long intervals of time in the ſame region of the heavens, and moving in the ſame curve, it is decided that they revolve about the Sun in very long or eccentric ellipſes; being governed throughout by the ſame law of deſcribing equal areas in equal times, which is found to take place in the inferior part of their orbits. The comet that appeared in the year 1661 was ſeen before in the ſame orbit, and under the ſame circumſtances in the year 1532; which ſhews its period to be 129 years: if it re-appears in the year 1790, its identity will be yet further confirmed. So likewiſe, the comet that appeared in the years 1456, 1531, 1607, 1682, and 1759, is determined to revolve in a period of about ſeventy-ſix years. And that very remarkable comet which was obſerved in the year 1680, is ſhewn to be the ſame with that which appeared in the year 1106; its period being 575 years. The diſtance of this comet from the Sun, when in its perihelium, was to the diſtance of the Earth from the ſame, in round numbers, as 6 to 1000: its heat therefore at that time was to the heat of the ſummer's ſun with us, as 1000000 to 36, or as 28000 to 1. But the heat of boiling water is about three times the heat which dry earth acquires from the ſummer's ſun; and the heat of red-hot iron may be about 3 or 4 times as great as that of boiling water. And therefore, the heat which the ſurface of the Comet would have acquired on the ſuppoſition of its being compoſed of dry earth, is about 2000 times greater than that of red-hot iron. By ſo fierce a heat, there is no doubt but vapours, exhalations, and every volatile matter muſt have been immediately conſumed and diſſipated.

The number of comets is very much greater w than that of the planets which move in the vicinity of the Sun. From the reports of former hiſtorians, as well as from the obſervations of late years, it is aſcertained, that more than four hundred and fifty have been ſeen previous to the year 1771: and when the attention of aſtronomers was called to this object by the expectation of the return of the comet of 1759; no fewer than ſeven were obſerved in the courſe of ſeven years. From this circumſtance, and the probability that all the comets recorded in ancient authors were of conſiderable apparent magnitude, while the ſmaller were overlooked, it is reaſonable to conclude, that the number of comets is conſiderably beyond any eſtimation that might be made from the obſervations we now poſſeſs. But the number of comets whoſe x orbits are ſettled with ſufficient accuracy to aſcer-

tain their identity when they may appear again is no more than fifty-nine, reckoning as late as the
Y year 1771. The orbits of moſt of theſe are inclined to the plane of the ecliptic in large angles, and the greater number of them approached nearer
Z the Sun than the Earth ever does. Their motions in the heavens are not all in the order of the ſigns, or direct, like thoſe of the planets: but the number whoſe motion is retrograde is nearly equal to that of thoſe whoſe motion is direct.

A It is not neceſſary in this work, to enter fully into the conſideration of final cauſes; more particularly as the ſubject has embarraſſed the greateſt metaphyſicians, and may with juſtice be ſaid to be too extended for the human powers. In every thing we ſee, the phenomena conſidered ſingly, are neceſſary conſequences of certain general laws, to which the univerſe appears to be ſubjected: but when they are conſidered in a collective view, a certain relation, or fitneſs for producing ſome general effects, is ſeen, which by no means depends on the ſame laws, and by analogy is referred to the operation of an intelligent agent. To illuſtrate this by example, we may obſerve, that in the well-known inſtrument called ſciſſars, it follows neceſſarily from the laws of motion, already explained in the mechanical powers (60, G. 67, H), that the blades will cut or divide certain ſubſtances expoſed to their action; but if we conſider the various circumſtances that co-operate in producing this effect, we muſt diſclaim all reaſoning

ing from analogy before we can reſolve their connection into an effect of thoſe laws, without ſuppoſing the agency of an intelligent being as the cauſe of their union, and concluding that it was intended they ſhould jointly concur in one purpoſe. It is to this being that we refer, in order to decide, why the ſharp edges were made on the inner, rather than on the outer, part of each blade: why the other extremities have annular terminations: why the inſtrument is made of ſteel rather than lead; and ſo forth. The purpoſes or motives which determine the actions of intelligent beings, and produce their effects in a manner ſimilar to the operation of the laws of nature or properties of matter in caſes where thought is not ſuppoſed to be concerned, are called final cauſes. In the works of nature we behold enough of ex- B
quiſite contrivance, and can ſee far enough into many final cauſes to convince us that the arrangement of the univerſe has been made, and probably is ſtill occaſionally adjuſted by a being whoſe intelligence and power is immenſely beyond that which we poſſeſs. To judge properly of his in- C
tentions, or in other words, to be equal to the taſk of exploring the ſcience of final cauſes, requires no leſs than a perfect knowledge and recollection of every purpoſe to which the objects around us may be applied, together with a clear conception of the ideas of fitneſs and order that form the prototypes in the mind of that great being who directs their motions. Theſe conſiderations ſhew the abſurdity D

of attempting to explain the final cauſes of every event we ſee, but they by no means require that we ſhould neglect them in caſes where we have reaſon to believe we underſtand the phenomena, and have ſufficient experience to be aſſured that we diſcern the principal, or at leaſt one of the principal purpoſes to which things may have been deſtined. Thus, it is ſcarcely to be imagined that we can err in concluding, that the eyes, ears, legs, wings, and other parts of animals were made for the purpoſes of ſeeing, hearing, walking, flying, and ſo forth. Neither can we avoid inferring, that the power who conſtructed living creatures with mouths, teeth, and organs to digeſt and aſſimilate food for their nutriment, did likewiſe form other organized bodies, which we call vegetables, for the expreſs purpoſe of affording that food. It is needleſs to multiply inſtances. We cannot avoid ſeeing them every moment, and their effect is ſo ſtriking, that we are inſenſibly forced from analogy to allow the exiſtence of a final cauſe in all caſes, whether we are able to diſcover it or no.

On this ground, an enquiry into the final cauſes of the planetary bodies offers itſelf to our conſideration. The earth is ſhewn to be a planet in circumſtances very ſimilar to the other five: we know its final cauſe—to ſupport a number of inhabitants. And by analogy, we may conclude, that the others are alſo habitable worlds; though from their different proportions of heat it is credible, that beings of our make and temperature could not live upon them. However, even that can ſcarcely be affirmed of all the planets: for the warmeſt climate on the planet Mars is not colder than many parts of Norway or Lapland are in the ſpring or autumn. Jupiter, Saturn, and the Georgium Sidus, it muſt be granted, are colder than any of the inhabited parts of our globe. The greateſt heat on the planet Venus, exceeds the heat in the iſland of St. Thomas on the coaſt of Guinea, or Sumatra, in the Eaſt Indies, about as much as the heat in thoſe places exceeds that of the Orkney iſlands, or that of the city of Stockholm in Sweden: therefore, at 60 degrees north latitude on that planet, if its axis were perpendicular to the plane of its orbit, the heat would not exceed the greateſt heat of the Earth, and of courſe, vegetation like ours might be there carried on, and animals of the ſpecies on Earth might ſubſiſt. If Mercury's axis be ſuppoſed to have a like poſition, a circle round each pole of about 20 degrees diameter would enjoy the ſame temperature as the warmer regions of the Earth, though in its hotteſt climate water would continually boil, and moſt inflammable ſubſtances would be parched up, deſtroyed, or diſſipated into vapor. But it is not at all neceſſary that the planets ſhould be peopled with animals like thoſe on the Earth: the Creator has doubtleſs adapted the inhabitants of each to their ſituation.

From the obſervations that have juſt been made, a better idea may be formed of the proportions of heat on the planets than can be conveyed by numbers. It will not however be remote from our purpoſe to compare the light of the ſuperior planets with that of our day, from whence it will appear, that they are by no means in a ſtate of darkneſs, notwithſtanding their great diſtance from the Sun. This might be inſtanced by ſeveral different methods, as by the Sun's light admitted into a dark chamber, and received on paper with different degrees of obliquity; by a greater or leſs number of candles brought into a room for the purpoſe of illuminating it with different proportions of light; or by vatious optical methods that need not be here deſcribed. It will be ſufficient for the illuſtration of the ſubject to compare their different proportions of light with that of a moonſhine night at the time of the full.

G When the Moon is viſible in the day-time, its light is ſo nearly equal to that of the lighter thin clouds, that it is with difficulty diſtinguiſhed amongſt them. Its light continues the ſame during the night; but the abſence of the Sun, ſuffering the aperture or pupil of the eye to dilate itſelf, renders it more conſpicuous. It therefore follows, that if every part of the ſky were equally luminous with the Moon's diſc, the light would be the ſame as if, in the day-time, it were covered with the thin clouds above-mentioned. This day-light is conſequently in proportion to that of the Moon, as the whole ſurface of the ſky, or viſible hemiſphere, is to the ſurface of the Moon; that is to ſay, nearly ly as 90,000 to 1. The light of the Georgium Sidus being to that of the Earth as 0.276 to 100, will be equal to the effect of 248 full Moons. The light of Saturn will be equal to that of 990 full Moons; Jupiter's day will equal the light of 3330 Moons, and that of Mars will require 38,700, a number ſo great, that they would almoſt touch one another. It is even probable, that the Comets, in the moſt diſtant parts of their orbits, enjoy a degree of light much exceeding moonſhine.

If the Comets be habitable, they muſt be poſſeſſed by creatures very different from any we have been uſed to behold and conſider. There are, however, many other uſes for which it is probable they may have been formed: the matter that compoſes their tails muſt fall in proceſs of time to the Sun or the neareſt planet that may paſs through it, where it may ſupply defects, and anſwer purpoſes which our total ignorance of its properties ſcarcely allows us even to conjecture. In the Sun it may ſerve to recruit the waſte of matter that luminary muſt ſuffer by the conſtant emiſſion of the particles of light. After a great number of revolutions, the reſiſtance of the Sun's atmoſphere, and a concurrence of circumſtances may occaſion the comet itſelf to approach the Sun, and at length fall into it, and become a part of its body.

CHAP. XI.

OF THE TELESCOPIC APPEARANCE OF THE MOON.

K THE obſervations which might confirm the hypotheſis of planetary worlds ſeem to be placed beyond our power. We can ſcarcely hope to make optical inſtruments ſufficiently perfect to render their inhabitants viſible to us. The groſs air that ſurrounds us, is a great impediment to the uſe of thoſe we already poſſeſs, and limits their perfection to a certain degree, beyond which we cannot paſs. All, therefore, that we can do, is to examine if the planets are accommodated with thoſe things which we are uſed to conſider as neceſſary to animal exiſtence. Lands, ſeas, clouds, vapours, and an atmoſphere or body of air, are objects that we may expect to find on the face of a habitable world: it is our preſent buſineſs to relate what has been done in this reſpect.

L The Moon being ſo very near us, and likewiſe in the ſame temperature as to light and heat, offers itſelf as the fitteſt body for examination. We diſcern a variety of ſpots with the naked eye, which the imagination naturally ſuppoſes to be ſeas, continents, and the like; but on a more accurate inſpection, with the aſſiſtance of the teleſcope, it is perceived, that many of thoſe appearances are occaſioned by vaſt obſcure pits or cavities,

ties, and elevations or mountains. The heights of theſe mountains may eaſily be found; for by the horizontal parallax we know that the Earth's apparent diameter, ſeen from the Moon at its mean diſtance, is 1° 54′ (132, x) or 6840 ſeconds, while that of the Moon ſeen from the Earth at the ſame diſtance is 31′, 2″9, or 1889 ſeconds. Their abſolute dia- M
meters muſt therefore be in proportion to theſe numbers. Conſequently, if we find the proportion the height of a lunar mountain bears to the Moon's diameter, we may, without difficulty, find the quantity of that height in miles or other terreſtrial dimenſions.

Theſe mountains and cavities are known to be N
ſuch from their ſhadows. In the firſt and ſecond quarters, when the Sun ſhines obliquely on the face of the Moon, the elevated parts caſt a triangular ſhadow in the direction from the Sun; and, on the contrary, the cavities are dark on the ſide next the Sun, and illuminated on the oppoſite ſide. The ſhadows ſhorten as the Sun becomes more directly oppoſed to the anterior face of the Moon, and at length diſappear at the time of the full. During the third and laſt quarters, the ſhadows appear again, but all fall towards the contrary ſide of the Moon, though ſtill with the ſame diſtinction, namely, that the mountains are dark and ſhady on the ſide furtheſt from the Sun, and the pits are dark on the ſide next the Sun. The ſame deduction is obtained by contemplating the inner illuminated edge of the Moon. If the Moon were a perfect ſphere, this

edge would be a regular curve, but if its ſurface be diverſified by hills and cavities, it is evident that the higher parts muſt be enlightened ſooner, and the cavities later than the reſt of the ſurface. This is accordingly the caſe, and affords a method of obtaining the heights of the mountains.

To render the explanation eaſier, we ſhall ſuppoſe the Moon to be in its quadrature, and the mountain to be ſituated near the Moon's apparent center.

Let the circle A B D G (fig. 61.) repreſent the Moon, whoſe center is C; and E the Earth: then a ſpectator at E will ſee the Moon enlightened in the half A G D, and the line E C will paſs through A, or the inner enlightened edge. The ray of light S A B touching the Moon at A, will croſs the line E C at right angles, and illuminate the top of the mountain B. The angle A E B is found by obſervation, then, in the triangle A E B,

As the co-ſine of the obſerved angle - A E B
Is to the Moon's diſtance - - A E,
So is the ſine of the obſerved angle - A E B,
To the ſide or line - - A B,

Then in the right angled triangle C A B, the ſides C A and A B being known, the ſide C B is found from the well known property (Euclid I. 47.); that is to ſay, the ſquare of the Moon's ſemidiameter C A being added to the ſquare of the line A B, the ſquare root of the ſum is the ſide C B. And if the ſemidiameter of the Moon C F be taken from the line C B, the remainder is F B, or the height of the mountain.

From obſervations of this kind, it appears that the lunar mountains are much higher in proportion to its radius than any we have upon the Earth. P

That the Moon is ſurrounded by an atmoſphere or body of air, is rendered probable by many obſervations of ſolar eclipſes, in which the limb or edge of the Sun was obſerved to tremble juſt before the beginning. The planets likewiſe are obſerved to change their figure from round to oval juſt before the beginning of an occultation behind the Moon; which can be attributed to no other cauſe, than that their light is refracted, being ſeen through the Moon's atmoſphere. Many aſtronomers are of opinion, that the Moon has no atmoſphere, becauſe we ſee no clouds, and becauſe the fixed ſtars diſappear at once at the time of an occultation without any gradual diminution of light, which they ſuppoſe ought to take place. But if we conſider the effect of days and nights near thirty times as long as with us, we may readily grant that the phenomena of vapours and meteors may be very different: perhaps their clouds and rain, if any, may be condenſed into viſible quantities, only during the abſence of the Sun, and if ſo, it is no wonder that we never ſee them. With reſpect to the fixed ſtars, it is plain, that granting the Moon to have an atmoſphere of the ſame nature and quantity as ours, no ſuch effect as a gradual diminution of light ought to take place, at leaſt as to ſenſe. Our atmoſphere is found to be ſo rare at the height of 44 miles, as to be incapable of acting on the Q rays of light. This height is the 180th part of the Earth's diameter; but ſince clouds are never obſerved higher than four miles, we muſt conclude, that the vaporous or obſcure part is but the 1980th part. The mean apparent diameter of the Moon is 31′ 29″, or 1889 ſeconds; therefore, the obſcure, part of its atmoſphere, when viewed from the Earth muſt ſubtend an angle of leſs than one ſecond, which ſpace is paſſed over by the Moon in leſs than two ſeconds of time; a ſpace and time ſo ſhort, that it can hardly be expected that obſervation can in general determine whether the ſuppoſed obſcuration takes place or not.

R It muſt however, be allowed, that the Moon's atmoſphere, if it has one, is certainly much leſs conſiderable than that of the Earth. For there is ſeldom any appearance, in an eclipſe of the Sun, of the remarkable phenomenon (158, Q), that the Earth's atmoſphere is ſhewn to produce in a lunar eclipſe*.

S The Moon turns round on its own axis once in the time of its periodical revolution. This is evident, becauſe the ſame face or ſide is conſtantly turned towards us. For a ſpectator on the Moon will ſee the Earth carried through every part of the ecliptic in the courſe of one revolution; and as the ſame face of the Moon is conſtantly turned towards the Earth, it muſt be ſucceſſively turned to every part of the ecliptic to which the Earth apparently moves.

---

* See Philoſophical Tranſactions, Anno 1779, for an account of the ſolar eclipſe of June 24, 1778, in which this appearance was ſeen by Don Antonio Ulloa.

Wes

UNIV. OF MICH.

moves. But if it be ſucceſſively turned to every part of a great circle in the heavens, it muſt revolve on its axis. By this ſlow rotation, it appears, that the inhabitants of the Moon have but one day and night in the courſe of a month.

T This rotation on its axis is the moſt uniform motion the Moon has; but its uniformity occaſions a ſeeming irregularity, which is termed the libration. For as the Moon's motion in its orbit was ſhewn to be not uniform (143, F) the effect it has in turning its face from the Earth is likewiſe ſubject to the ſame irregularities; for inſtance, in the ſwifteſt part of the revolution, its motion in its orbit turns its face from the Earth ſomething more than the rotation on its axis turns it the other way, and therefore it appears to have a ſmall motion on its axis towards the eaſt, by which ſome of the more weſtern parts are brought into view, and an equal portion of the eaſtern limb diſappears. In the ſlower part the contrary is ſeen, for then the rotation on its axis prevailing, brings the weſtern parts into view, and the eaſtern diſappear. This is called libration in longitude.

U There is another kind of libration that ariſes from the Moon's axis being inclined to the plane of its orbit, by which means ſometimes one of its poles, and ſometimes the other, is inclined towards the Earth. In conſequence of this, we ſee more or leſs of the polar regions at different times. This is called libration in latitude.

CHAP. XII.

OF THE TELESCOPIC APPEARANCES OF THE SUN AND THE PLANETS.

THE Sun is not without ſpots on its diſc, but they are ſeldom ſo large as to be ſeen by the naked eye. When viewed with a teleſcope, the eye being defended by a piece of coloured or ſmoked glaſs, they are found to appear in various forms and numbers. The larger ſpots, moſt of which exceed the whole Earth in apparent magnitude, laſt a conſiderable time; ſometimes three months before they diſappear, at which time they are generally converted into faculæ, or ſpots which exceed the reſt of the Sun in brightneſs. They are of no conſtant figure, frequently changing during the time of obſervation, and ſometimes one dividing into ſeveral ſmaller ones. In general they conſiſt of a nucleus or central part, much darker than the reſt, which is ſurrounded by a miſtineſs or ſmoke. The general opinion concerning them is, that they are occaſioned by the ſmoke and opake matter thrown out by volcanos or burning mountains of immenſe magnitude; and that when the eruption is nearly ended, and the ſmoke diſſipated, the fierce flames are expoſed, and appear as faculæ, or luminous ſpots. But Dr. Alexander Wilſon of Glaſgow has eſtabliſhed from obſervation, that moſt, if not all the ſpots, are excavations in the luminous matter that environs the body of the Sun, probably to no great depth*. At preſent (anno 1779) they are often ſeen to the number of thirty or more, but there have been periods of more than ſeven years, in which none have been obſerved.

All the ſpots of the Sun have an apparent motion w
from eaſt to weſt, which is quicker when they are near the central regions than when near the limb. This proves that the Sun revolves on its axis from x
weſt to eaſt, and likewiſe that its figure is ſpherical. The period, as obſerved by Caſſini, is 25 days, 14 y
hours, 8 minutes. From the line of the motion of the ſpots, which is ſometimes ſtrait, but oftener curved or elliptical, it is diſcovered that its axis is z
not perpendicular to the ecliptic, but inclined, ſo as to make an angle with the perpendicular of about ſeven degrees and a half.

The zodiacal light is attributed to the ſolar atmo- A
ſphere. This remarkable phenomenon accompanies the Sun. When it begins to appear before ſunriſe, it ſeems at firſt ſight like a faint, and almoſt imperceptible, whitiſh light, reſembling the milky-way, and ill-terminated, which is almoſt confounded with the twilight that is ſeen commencing near the horizon. It is then little elevated, and its termination may ſometimes be diſcerned in a conic or conoidal form. In ſhort, its figure agrees with that of a very flat or lenticular ſpheroid ſeen in profile. As it gradually riſes above the horizon it becomes brighter, and larger to a certain point, that may be cal-

---

* Philoſophical Tranſactions, 1774.

led its maximum, after which the approach of day renders it gradually leſs apparent, and at laſt inviſible. The direction of its longer apparent axis is obſerved to be in the plane of the Sun's equator; but its length is ſubject to great variation, ſo that the diſtance of its ſummit from the Sun varies from 45° to even 120°. Theſe great differences in magnitude and brightneſs may perhaps depend conſiderably on ſome, yet unknown, circumſtances in our atmoſphere. It is uſually ſeen to the greateſt advantage about the ſolſtice.

B It has been ſuppoſed, that this atmoſphere is the cauſe of the aſcent of the vapor which forms the tails of the comets, and which is always carried to that part of the heavens which is oppoſite the Sun. But the direction of theſe vapors may perhaps be determined by the action of the particles of light emitted from the Sun.

C The planet Mercury is at all times ſo near the Sun, that we can only diſtinguiſh with the teleſcope a variation in its figure, which is ſometimes that of a half Moon, and ſometimes a little more or [illegible] than half. Whence it follows, that its form is globular, and that it receives all its light from the Sun.

D The planet Venus, when viewed through the teleſcope, has a very pleaſing appearance. At the time of its greateſt elongation it appears like the Moon in the quadratures; one half of its diſc being enlightened. In the inferior part of its orbit, as its elongation decreaſes, the enlightened part becomes leſs, appearing falcated or horned; after paſſing the inferior conjunction, the planet is again ſeen horned, but the illuminated part then increaſes, and at the greateſt elongation, half its diſc is again ſeen enlightened. In the ſuperior part of its orbit, as its elongation decreaſes, its face becomes more full and round, till the ſuperior conjunction, after which time it is again diminiſhed by the ſame gradation as its increaſe was in the former caſe accompliſhed. There is no difficulty in accounting for this variety of phaſes, it being occaſioned by the different poſitions of Venus with reſpect to the Sun and Earth: for as the enlightened face of Venus muſt of courſe be always oppoſite to or facing the Sun, it will be more or leſs viſible to us according to our ſituation at various times.

The ſurface of Venus is diverſied with ſpots like our Moon, by the motion of which it is determined, that it revolves on its axis from weſt to eaſt in the ſpace of twenty-three hours. When the air is in a good ſtate for this kind of obſervations, mountains like thoſe in the Moon may be diſcerned, with a very powerful teleſcope.

The face of the planet Mars is always round and full, as its ſuperior ſituation requires, excepting at the time of the quadrature, or elongation of 90 degrees, when a ſmall part of the unenlightened hemiſphere being turned towards us, its diſc appears like the Moon about three days after the full.

By the ſpots on Mars; its diurnal revolution is aſcertained in the direction from weſt to eaſt.

From the ruddy and obſcure appearance of this planet, as well as from other appearances, it is concluded, that its atmoſphere is nearly of the ſame denſity as that of the Earth. Mr. Herſchel has obſerved, that two circles ſurrounding the poles of this planet are very white, and luminous, probably, from ſnow lying there.

H We have already had occaſion to ſpeak of the ſatellites of Jupiter and Saturn. The annual parallax of theſe planets is not conſiderable enough to bring any ſenſible part of their dark hemiſpheres towards us in any poſition of elongation; conſequently their faces are always round and full.

I The teleſcopic appearance of Jupiter affords a vaſt field for the curious enquirer. It is in general encircled with one or more obſcure belts or bands parallel to the plane of its orbit, and conſequently to each other. Theſe are not regular or conſtant in their appearance. They have been ſeen to the number of five, and during the time of obſervation two have gradually diſappeared. Sometimes but one is ſeen; and ſometimes, when the number is more conſiderable, one or more dark ſpots are formed between the belts, which increaſe till the whole is united in one large duſky band. The ſpots of Jupiter are the brighter parts of its ſurface, and are not permanent, though more ſo than the belts; yet it is found that they re-appear after certain unequal intervals of time. The remarkable ſpot, by whoſe motion the rotation of Jupiter on its axis was determined, diſappeared in 1694, and was not ſeen again till 1708, when it re-appeared exactly in the ſame place on its ſurface, and has been occaſionally ſeen ever ſince.

K It has been conjectured, that theſe belts are ſeas, and that the variations obſerved both in them and the ſpots are occaſioned by tides, which are differently affected, according to the poſitions of his moons. If an obſerver, poſſeſſed of ſkill and patience equal to the taſk, would delineate the phaſes of Jupiter for the ſpace of a periodical revolution, noting at the ſame time the poſitions of his ſatellites, this opinion might be either eſtabliſhed or rejected: but at all events ſuch a ſeries of obſervations could not fail to throw great light on the ſubject.

L The very great diſtance of the planet Saturn, and the tenuity of its light, do not permit us to diſtinguiſh thoſe varieties which it is probable are on its ſurface. The faint appearance of a belt is ſometimes ſeen. The ring which encircles this planet is inclined to the ecliptic; in conſequence of which, its apparent figure is continually varying. When the line of its nodes points directly towards the Earth, the ring, preſenting its edge to the obſerver, becomes inviſible: if the ſame line points directly towards the Sun, the ring becomes inviſible for want of illumination: and laſtly, if the plane of the ring paſſes between the Sun and the Earth, the ring cannot then be ſeen, becauſe its dark ſide is towards us. At all other times its figure is that of an oval, which is broader or narrower accordingly as the line of the nodes is farther from or nearer to the above poſitions.

CHAP. XIII.

OF THE LENGTH OF DAYS AND NIGHTS; AND OF THE SEASONS.

M WE have ſeen that every planet which is acceſſible to obſervation has a revolution on its axis; the intention of which is, undoubtedly, to give alternate night and day to every part of their ſurfaces. An inclination of the axis of any planet to the axis of its orbit, by cauſing the length of days and the intenſity of heat to vary, will occaſion a viciſſitude of ſeaſons. On this account Jupiter, whoſe axis is nearly perpendicular to the plane of its orbit, has equal days and nights on every part of its ſurface at the ſame time, the days being four hours and twenty-eight minutes, and the nights of the ſame length. But the planets Mars and Venus, whoſe axes are inclined to the planes of their reſpective orbits, have each an annual change of ſeaſons and length of days. The Earth, for the ſame reaſon, has a ſimilar viciſſitude, the explanation of which will render it unneceſſary to enlarge on the circumſtances of the other planets.

N For this purpoſe it will be uſeful to define thoſe imaginary circles, which aſtronomers and geographers have invented for the purpoſes of methodizing and facilitating the communication of ſcience.

O On the Earth, a great circle, ſuppoſed to be drawn at an equal diſtance from each pole, is termed the Equator: leſs circles drawn parallel to the equator are called Parallels of Latitude; and great circles interſecting the equator at right angles, and conſequently paſſing through the poles, are called Meridians. But when the meridian of a place is ſpoken of, it is uſually underſtood to be a ſemicircle paſſing through the given place, and terminating at the poles. The other half which completes that whole circle, is then called the oppoſite meridian.

P In the heavens, a great circle, parallel to the equator, is termed the Celeſtial Equator; but the leſs circles parallel to it are called Parallels of Declination; and the great circles interſecting it at right angles, and paſſing through the celeſtial poles of the Earth, are called Hour Circles, or circles of right aſcenſion.

Q The ecliptic is that great circle in the heavens, in which the Sun deſcribes its apparent annual courſe: leſs circles, drawn parallel to the ecliptic, are called Parallels of Latitude; and great circles interſecting it at right angles, and conſequently paſſing through its poles, are called Celeſtial Meridians.

R The horizon is that great circle which divides the viſible or upper hemiſphere from the lower. If this circle have the eye of the obſerver for its center, it is called the Senſible Horizon; but if its center be that of the Earth, it is termed the Rational Horizon. To this laſt all aſtronomical obſervations are reduced or applied; the former being only conſidered as one of the parallels of altitude. Leſs circles, parallel to the horizon, are called Parallels of Altitude, if above, but of Depreſſion, if below the horizon, and the great circles interſecting it at right angles, are called Azimuths.

S The point of the heavens, which is immediately above the obſerver, or is elevated 90° above the horizon, is termed the Zenith; the oppoſite point immediately beneath, or at 90° of depreſſion below the horizon, is termed the Nadir.

T Latitude on the Earth is an arc of the meridian, contained between a given place and the equator. It is meaſured in degrees and minutes of the meridian. In the heavens it is an arc of the celeſtial meridian, contained between a given place and the ecliptic.

U Longitude on the Earth is an arc of the equator, contained between the meridian paſſing through a given place and the firſt meridian. It never exceeds a ſemicircle. The firſt meridian on the Earth is arbitrary; but the Engliſh aſtronomers in general reckon from that which paſſes through the obſervatory at Greenwich. Longitude in the heavens is an arc of the ecliptic, contained between a given meridian and that which paſſes through the firſt point of the conſtellation Aries; the ſaid point being always at the weſtern extremity of the arc.

V Right aſcenſion is an arc of the celeſtial equator, contained between a given hour circle and that which paſſes through the firſt point of the conſtellation Aries; the ſaid point being always at the weſtern extremity of the arc. Declination is an arc of the hour circle, contained between a given place and the equator.

The circle which divides the enlightened hemiſphere of a planet from its dark hemiſphere is called the Terminator. It may in moſt caſes be conſidered as a great circle. W

Let N E S Q (fig. 62. and 63.) repreſent the Globe of the Earth, and C the Sun: then the circles N M S, N M S, &c. will repreſent the meridians interſecting the Equator E Q at right angles, and paſſing through the poles N and S. The lines pp, pp, &c. will repreſent the parallels of latitude; and the line C M will repreſent the plane of the Earth's orbit. X

Now it is evident, that it is day at any given place on the globe, ſo long as that place continues in the enlightened hemiſphere; and that when by the diurnal rotation it is carried into the dark hemiſphere it becomes night; twilight not being here conſidered. And from the contemplation of figure 63, it appears, that if the poles be ſituated in the terminator, the terminator will divide each of the parallels into two equal parts, and conſequently, ſince the uniform motion of the Earth cauſes any given place to deſcribe equal parts of its parallel in equal times, the days and nights will be equal on every parallel of latitude; that is to ſay, all over the globe, except at the poles, where the Sun will neither riſe nor ſet, but continue in the horizon. Y

But if, as in figure 62, the axis be not placed in the plane of the terminator, the terminator will divide the equator into two equal parts; but the parallels which are ſituated towards the enlightened pole will have a greater part of their peripheries in Z the enlightened than in the dark hemiſphere: while ſimilar parallels towards the other pole will have a like greater part of their peripheries in the dark hemiſphere. Whence it follows, that the firſt-mentioned parallels will enjoy longer days than nights, and the contrary will happen to the latter, they having ſhorter days and longer nights; while at the equator the days and nights continue equal. All this is evident from the figure, where it is alſo obſervable, that the diſproportion is greateſt in the greater latitudes; and that places, whoſe diſtance from the pole is leſs than that of the pole from the terminator, muſt enjoy either a conſtant day or conſtant night, the rotation of the Earth never carrying them into the oppoſite hemiſphere.

A In this poſition of the axis the inhabitants on the one ſide of the equator may be ſaid to enjoy ſummer, and thoſe on the other ſide winter with reſpect to each other. For the long duration of the Sun above the horizon muſt occaſion a proportionally greater degree of heat, and its longer abſence from places ſituated in the other hemiſphere muſt have the contrary effect.

B But this is not the only cauſe of the difference of heat at the different places. A ſpectator at G, which is 90° diſtant from the terminator, will have the Sun in the zenith; a ſpectator at T will ſee the Sun in the horizon; and, for every intermediate diſtance, the arc of a great circle comprehended between the terminator and the place of obſervation will be the meaſure of the Sun's altitude. Therefore every parallel between Q and the enlightened pole will have the meridian altitude of the Sun increaſed, by the angle N M T, beyond what it would have been had the pole continued in the plane of the terminator: and every place between Q and the dark pole will have the Sun's meridian altitude diminiſhed by the ſame quantity. Between G and Q the former altitude thus increaſed, making a ſum greater than 90°, the altitude muſt be meaſured by its complement. And between S and T the altitude will be a negative quantity, or beneath the horizon. This difference of the altitudes of the Sun muſt cauſe an increaſe of heat towards the enlightened pole, and an equal diminution towards the dark pole. For the greater the Sun's altitude, the more directly its rays fall on any ſurface; and c in ſurfaces of the ſame magnitude the quantity of light received by each is as the ſine of the angle of obliquity with which the rays fall *.

It remains to be ſhewn, that theſe relative ſitu- D ations of the axis and the terminator take place at different times of the year, with reſpect to the Earth;

---

* Let the line A B (fig. 64.) repreſent a ſurface, on which the column of light N O A B falls perpendicularly. A ſurface A C, of the ſame magnitude, receiving the light obliquely under the angle J C K, will intercept only ſo much as would have fallen on the ſpace A E and another ſurface A D, receiving the light ſtill more obliquely under the angle L D M, will intercept only ſo much as would have fallen on the ſpace A F, But the ſpaces or lines A E and A F are the ſines of the angles of obliquity J C K and L D M; whence the propoſition is evident.

Earth; which being proved, the viciſſitude of ſeaſons muſt follow as a neceſſary conſequence.

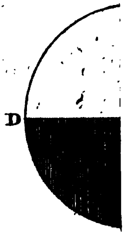
E In fig. 65. Let C repreſent the Sun, ABDE the Earth's orbit, nearly circular, but which being viewed obliquely, appears like a long ellipſis, of which let the part BD be ſuppoſed neareſt the ſpectator. And let the four circles, diſtinguiſhed by the months March, June, September, and December, repreſent the Earth in four ſeveral parts of its orbit, NS being its axis.

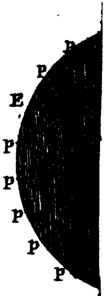
F Obſervation ſhews, that the axis of the Earth always preſerves very nearly the ſame poſition with reſpect to the fixed ſtars; being inclined to the axis of its orbit in an angle of about 23½ degrees. It will not therefore preſerve the ſame relative poſition with reſpect to the terminator. For when the Earth is in the ſituation diſtinguiſhed by the month March, its axis will at that time be in the plane of the terminator, and conſequently the days and nights will be equal all over the globe (183, Y): but when by its annual motion it is carried towards A, the north pole N, the axis ſtill preſerving its poſition or continuing parallel to itſelf, will advance into the enlightened hemiſphere, and in the month of June will be 23½ degrees diſtant from the terminator, as in the ſcheme, the ſouth pole being at the ſame diſtance in the dark hemiſphere. Therefore in the month of June the northern parts will enjoy long days and ſummer, while the ſouthern parts have ſhort days and winter (183, Z).

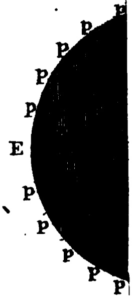
During the interval between the time of equal days and nights in March, which is called the vernal equinox, and the time when the day is longeſt in June, which is called the ſummer ſolſtice, the north pole will have deſcribed a quarter of a circle in the enlightened hemiſphere with reſpect to the terminator, and conſequently will be at its greateſt diſtance from it. From that time it will, by deſcribing the other quarter, approach the terminator, the days gradually ſhortening till the Earth arrives at the poſition denoted by the month September, when, the axis again coinciding with the plane of the terminator, the days and nights will be equal. This is called the autumnal equinox. During the next quarter the north pole will deſcribe a quarter of a circle in the dark hemiſphere, and the days will ſhorten till December, when the pole will be juſt as far within the dark as in June it was in the enlightened hemiſphere, which time is called the winter ſolſtice. From the winter ſolſtice to the vernal equinox, the days will lengthen as the pole approaches the terminator; and at the inſtant in which the axis again coincides with its plane, the natural year, conſiſting of 365 days, 5 hours, 48 minutes, and 45 $\frac{1}{2}$ ſeconds, is finiſhed.

It is eaſy to conceive, by applying the ſame explanation to the ſouth pole inſtead of the north, that the inhabitants of the ſouthern hemiſphere have the ſame viciſſitudes, though not at the ſame time; for it is winter in one hemiſphere while it is ſummer in the other, &c. &c.

1 As the pole N (fig. 62.) advances in the enlightened hemiſphere, the Sun will be in the zenith of a place G, as far diſtant from the equator as the pole is from the terminator; therefore the greateſt latitude at which the Sun can be vertical is 23½ degrees. The parallels of latitude on the Earth of 23½ degrees N. and S. as alſo the correſpondent parallels of declination in the heavens are called the Tropics, becauſe the Sun when it arrives at them afterwards returns towards the equator. The Sun, when it arrives at the northern tropic, is juſt entering the ſign Cancer, and when it arrives at the ſouthern tropic is juſt entering the ſign Capricorn; for which reaſon the northern tropic is called the Tropic of Cancer, and the ſouthern tropic the Tropic of Capricorn.







UNIV. OF MICH.

CHAP. XIV.

OF THE FIXED STARS.

THOUGH in a former chapter of this ſection it was mentioned (104.) that the relative ſituations of the fixed ſtars do not vary, yet that aſſertion is not to be underſtood in abſolute ſtrictneſs. In the courſe of ages ſeveral variations have been obſerved amongſt them. Some of the larger ſtars have not the ſame preciſe ſituations that ancient obſervations attribute to them; and it is probable that the inſtances of this kind would have been much more numerous if accuracy of obſervation had not been confined to a very late period. New ſtars have likewiſe appeared from time to time, and ſeveral of thoſe whoſe places and magnitudes are inſerted in the old catalogues are not now to be found. Some of the fixed ſtars are likewiſe found to have a periodical increaſe and decreaſe of magnitude. K

The bright ſtars, Arcturus, Sirius, Aquila, and Aldebaran, have been obſerved to change their places. The firſt is found to move towards the ſouth, about 3¼ minutes of a degree in a century. Sirius has advanced about 2 minutes to the ſouth in a like period. The changes of place in the two latter are yet ſmaller and leſs ſettled. L

All the ſtars ſpoken of in the preſent chapter are ſubject to no parallax (130, z), according to the M the moſt accurate obſervations by which moſt of their places were ſettled; and ſome of them have been obſerved with inſtruments of ſuch delicacy, that it is preſumed their parallax would have been ſeen, if it had amounted to one ſecond of meaſure.

N Without attending either to the celeſtial changes recorded by ancient authors, who for the moſt part were not aſtronomers, and paſſing the leſs obvious mutations in ſilence, we ſhall here note a few of the moſt remarkable new or changeable ſtars.

O On the 8th of November, 1572, Cornelius Gemma attentively conſidered that part of the heavens which is called Caſſiope's Chair, and perceived nothing extraordinary. But the following night a new ſtar appeared, forming a perfect rhombus with the three ſtars α, β, γ, of that conſtellation. Its ſplendor exceeded that of Jupiter when greateſt, and was ſuch, that it was ſeen even in the day-time. Tycho Brahe, who ſaw it on the 11th, determined its longitude 6°. 54′ of Taurus, with 53°. 45′ N. latitude. It began to diminiſh in December, and became gradually leſs conſpicuous, till it diſappeared in March, 1574. This remarkable ſtar had no apparent motion, and conſequently no parallax, and its appearance was ſparkling and clear, like that of the other fixed ſtars. It has not ſince been ſeen.

P September, 1604, O. S. the ſcholars of Kepler obſerved a ſtar in the right leg of Serpentarius, which was not there the night before. Its luſtre ſeems to have been nearly equal to that of the new ſtar in Caſſiope; for it is deſcribed as exceeding Jupiter in brightneſs. It gradually decayed like that, and in nearly the ſame time diſappeared, not being perceived after the beginning of January, 160$\frac{5}{6}$. Its right aſcenſion, as obſerved by Kepler, was conſtantly 256° 57′, and its declination 21°01$\frac{1}{2}$′ S.

Q The firſt ſtar that was obſerved to have a periodical change of brightneſs was diſcovered by David Fabricius in the neck of the Whale, on the 3d of Auguſt 1596, O. S. Its greateſt brightneſs is equal to that of a ſtar of the third magnitude; and it is ſcarcely ever ſo ſmall but it may be ſeen with a ſix foot teleſcope. The period in which it paſſes through all its changes, is at a mean 334 days, but no part of the phenomenon is perfectly regular.

R Three changeable ſtars have been obſerved in the conſtellation of the Swan. The firſt diſcovered is near the ſtar γ in that conſtellation. Its greateſt luſtre is leſs than that of a ſtar of the third magnitude, and it diminiſhes to that of the ſixth magnitude. Its changes are far from being regular, and do not take place but after intervals of ten or more years.

S The next and moſt remarkable of the changeable ſtars in the Swan is marked χ by Bayer. This is more regular in its returns than the preceding, though its magnitude is ſeldom greater than the ſixth. Its period is ſettled at 405$\frac{3}{10}$ days, and its greateſt luſtre in the year 1785 will be about the 14th of July.

The

r The third was ſeen near the head of the Swan on the 20th of June, 1670, of about the third magnitude, and was ſo far diminiſhed by the October following as to be ſcarcely viſible. In the beginning of April, 1671, it was again ſeen rather brighter than before, and diminiſhing during that month, became once more at its greateſt brilliancy at the beginning of May. By a compariſon of theſe obſervations, its period ſeemed about ten months. It diſappeared about the middle of Auguſt, and was again ſeen on the 29th of March, 1672; ſince which time it has not appeared.

u The ſtar Algol, or Meduſa's head, has been long ſince obſerved to appear of different magnitudes at different times; but the diſcovery of its period is due to John Goodricke, Eſq; of York, who has obſerved it ſince the beginning of the year 1783. It periodically changes from the firſt to the fourth magnitude; and the time employed from one greateſt diminution to the other, was Anno 1783, at a mean, 2 days, 20 hours, 49 minutes, 3 ſeconds. The change is thus. During four hours it gradually diminiſhes in luſtre; during the ſucceeding four hours it recovers its firſt magnitude by a like gradual increaſe; and during the remaining part of the period, namely, 2 days, 12 hours, 49 minutes, 3 ſeconds, it invariably preſerves its greateſt luſtre: after the expiration of which term the diminution again commences, &c.

v Many of the fixed ſtars, upon examination with the teleſcope, are found to conſiſt of two. The number obſerved before the time of Herſchel * was but ſmall; but that celebrated aſtronomer, who ſtands unrivalled for the excellence of his inſtruments, and his ſkilful induſtry in uſing them, has noted upwards of four hundred.

w Beſides the phenomena already mentioned, there are many nebulæ, or parts of the heavens which are brighter than the reſt. The moſt obvious to common notice is that large irregular zone or band of light which croſſes the ecliptic in Cancer and Capricorn, and is inclined to it in an angle of about ſixty degrees. Other nebulæ are ſeldom ſo large as to be ſeen by the naked eye, to which they appear as ſmall ſtars. If the teleſcope be applied to them, they ſeem to be luminous ſpots of various figures, in ſome inſtances with ſtars in them. The number of nebulæ aſcertained before Herſchel are about 103, and that obſerver had detected 466 more, previous to the month of April 1784. Many of the nebulæ are reſolvable by the teleſcope into cluſters of ſmall ſtars; and it is found that teleſcopes of greater power reſolve thoſe nebulæ into ſtars, which appear as white clouds in inſtruments of leſs force. Hence there is good reaſon to conclude that they all conſiſt of cluſters or prodigious aggregates of ſtars.

x Mr. Herſchel has rendered it highly probable, both from obſervation and well-grounded conjecture, that the ſtarry heaven is replete with theſe nebulæ or

---

* Mr. Herſchel's numerous and important diſcoveries are inſerted in the late volumes of the Philoſophical Tranſactions, as are alſo the accounts of Algol.

ſyſtems of ſtars of various figures, and that the milky-way is that particular nebula in which our Sun is placed. Nothing more is neceſſary in order to account for the appearance it exhibits, than to aſſume its figure as being much more extended towards the apparent zone of illumination than in other directions. And the obſervations on the various figures of nebulæ render this ſuppoſition perfectly allowable.

Y The want of an annual parallax in the fixed ſtars evinces, that a luminous body, whoſe diameter is equal to that of the Earth's annual orbit, would not ſubtend a ſenſible angle if ſeen from the fixed ſtars. Much leſs therefore would the Sun, if viewed from ſuch a diſtance. It may therefore be fairly concluded, that the Sun, when ſeen from any fixed ſtar, muſt have much the ſame appearance as a fixed ſtar ſeen from hence: or, in other words, the fixed ſtars are ſuns.

Z Reaſoning then analogically, as far as the nature of the facts we poſſeſs will admit, it may be deduced; firſt, that the univerſe conſiſts of nebulæ, or diſtinct ſyſtems of ſtars; ſecondly, that each nebula is compoſed of a prodigious number of ſuns, or bodies that ſhine by their own native ſplendor; thirdly, that each individual ſun is deſtined to give light to hundreds (124, R. 161, W. 164, E.) of worlds that revolve about it, but which can no more be ſeen by us, on account of their great diſtance, than the ſolar planets can be ſeen from the fixed ſtars.

Yet, as in this unexplored, and, perhaps, unexplorable abyſs of ſpace, it is no neceſſary condition that the planets ſhould be of the ſame magnitudes as thoſe belonging to our ſyſtem, it is not improbable but that planetary bodies may be diſcovered among the double and triple ſtars. A

Our curioſity is much intereſted in the contemplation of the phenomena of new and changeable ſtars, but the cauſes that may be offered with plauſibility to ſolve theſe appearances are not many. C

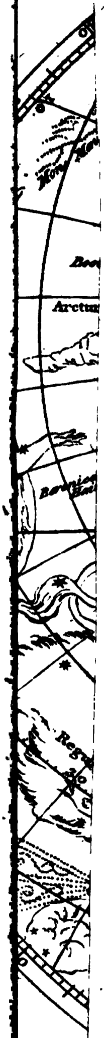
If the light of the Sun and fixed ſtars be imagined to proceed from a combuſtion ſimilar to that which is required to produce light in our experiments, it may happen that when all the inflammable matter is decompoſed the ignition may ceaſe. Or, if a maſs of matter adapted for inflammation begin by any cauſe to burn, its ignition and emiſſion of light will then commence. Theſe conſiderations may explain the diſappearance of ſome ſtars, and the appearance of others. And as there are no data to fix the time between the beginning and end of the appearance, the ſtars may laſt for any given time, according to circumſtances. E

The ſpots on the Sun (174, v.) have afforded a conjecture reſpecting the cauſe of the periodical change of brightneſs in ſome ſtars. For, if a ſtar be ſuppoſed to have a ſpot of conſiderable magnitude, and to turn on its axis, it will be much brighter when the ſpot is not on the viſible diſc than when it is. However, it muſt be confeſſed, that the phenomena do not in general agree with this ſuppoſition, D which cannot eaſily be reconciled to the permanent brightneſs or obſcurity that prevails in the changeable ſtars for more than half the period.

E If a ſtar, by a ſwift revolution, be made to aſſume and preſerve a flattened figure, and its axis have a rotation ſimilar to that hereafter to be explained in the Earth, it will be much leſs bright when its edge is preſented to the obſerver than when the viſible diſc is projected broader.

F Or, laſtly, if a large planet revolve about a ſtar, it may occaſion certain periodical eclipſes of ſufficient magnitude and duration to be perceived by us, on account of the quantity of light intercepted.
G Thus, for example, if an opake planet, whoſe diameter is not much leſs than that of Algol, be ſuppoſed to revolve about that ſtar at the diſtance of thirty-three diameters of Algol, in the given period of 2 days, 20 hours, 49 minutes, 3 ſeconds, in an orbit whoſe plane paſſes at preſent through or near the Earth, it will cauſe certain eclipſes that will agree very well with the appearances obſerved.

BOOK



UNIV. OF MICH.

UNIV. OF MICH.

BOOK I.

SECT. IV.

Concerning the phyſical cauſes of the celeſtial motions.

CHAP. I.

OF THE GENERAL EFFECTS OF CENTRIPETAL FORCES ON BODIES IN MOTION.

WE are now to reſume the conſideration of bodies in motion, which are acted on by a centripetal force (94, I) and to apply that doctrine to the phenomena explained in the preceding ſection. A

Let A B C D, &c. (fig. 66.) be a regular polygon, inſcribed in a circle. If a body be ſuppoſed to revolve in that polygon, it will be neceſſary that a force directed to the center S ſhould be applied at the points B, C, D, &c. to deflect the body from its right lined courſe (21, P). The more numerous the ſides of the polygon are, the nearer they will be to the circular curve, and the more frequent muſt be the ſucceſſive actions of the centripetal force. And if the number of ſides be infinitely great, the polygon will abſolutely coincide with the circle, or become a circle, B

and the actions of the centripetal force muſt be infinitely numerous, or the force will act without inter-
C miſſion. Conſequently, whatever may be proved in general of a body moving in a regular polygon by an original uniform motion, combined with the motion produced by the ſucceſſive actions of a force directed to the center of the polygon, will hold good with reſpect to the motion of a body in a circle, the centripetal force being ſuppoſed to act without intermiſſion.

D In our reaſoning concerning centripetal forces, it is here ſuppoſed that a given force acts on bodies according to their maſſes, like gravity (26, A) and conſequently cauſes equal deflections in each from the right lined courſe.

E The intenſity or quantity of any force is meaſured by the effect it produces in a given time (21, Q. 38, T). Suppoſe a body to be projected from M to A, fig. 66. a centripetal force repreſented by A R will cauſe it to deſcribe the line A B inſtead of A Q in an equal ſpace of time (23, T). If the velocity in MA had been greater, the action of the force in the line A R muſt have been greater in the ſame proportion to have cauſed the ſame deflection; that is, the centripetal force muſt at each point of deflection be as the velocity. But again, the greater the velocity the greater number of ſides of the polygon will be deſcribed by the body in a given time, and the more frequent muſt be the actions of the centripetal force. For this reaſon therefore, the number of the actions
F muſt likewiſe be as the velocity. On both accounts therefore,

therefore, the whole effect of the force in a given time, or its intenſity, muſt be in the duplicate proportion, or as the ſquare of the velocity, that is, ſimply as the velocity, becauſe the actions themſelves are greater or leſs in that proportion; and again, ſimply as the velocity, becauſe the actions take place more or leſs frequently in the ſame proportion. And the ſame is true of the G
effect of an unceaſing force that may cauſe the body to revolve in the circle in which the polygon is inſcribed (198, c).

The polygon N O P being ſimilar to the polygon H
A B C, will, with a given velocity, require the ſame action to cauſe the requiſite deflections from the right lined courſe in a body revolving in it. But thoſe actions muſt recur oftener in proportion as the ſide of the polygon N O P is leſs than that of the polygon A B C, becauſe a proportionally greater number of the ſmaller ſides will be paſſed over with the ſame velocity. The force in the ſmaller polygon muſt therefore be increaſed in the inverſe proportion of its ſide to that of the greater, or, which is the ſame, in the inverſe proportion of their radii. On the whole then, the I
centripetal forces, by which bodies are retained in circular orbits, (198, c) are in a ratio compounded of the direct ratio of the ſquares of the velocities, (199, F, G) and the inverſe ratio of their ſemidiameters.

The periodical times of bodies revolving in circles are greater, the greater the radii, and leſs, the [illegible] greater the velocities. That is to ſay, the periodical times are directly as the radii, and inverſely as the velocities.

L If the centripetal forces be ſuppoſed to increaſe as the cubes or third powers of the radii decreaſe, the cubes of the radii will be inverſely as the ſquares of the velocities, and directly as the radii, becauſe the forces themſelves are inverſely in this compound ratio (199, I). Whence the ſquares of the velocities will be directly as the radii, and inverſely as the cubes of the radii; or more ſimply, M as the ſquares of the radii inverſely. And the velocities themſelves will conſequently be inverſely as the radii.

N If the ſquares of the periodical times be directly as the cubes of the radii, the cubes of the radii will be (199, K) directly as the ſquares of the radii, and inverſely as the ſquares of the velocities. Therefore, the ſquares of the velocities will be directly as the ſquares of the radii, and inverſely as the cubes of the radii; or, more ſimply, as the radii inverſely. Now the centripetal forces (199, I) are directly as the ſquares of the velocities, and inverſely as the radii. Therefore, if in this laſt compound ratio we ſubſtitute the inverſe ratio of the radii, inſtead of the direct ratio of the ſquares of the velocities, to which it is equal, we ſhall have the forces in the inverſe ratio of the ſquares O of the radii. That is to ſay, if the ſquares of the periodical times of bodies revolving in circles be directly as the cubes of the radii, the centripetal forces will be inverſely as the ſquares of the radii.

The velocity of a body moving in a curve, and acted on by a centripetal force, is inverſely as the perpendicular let fall from the center, to the tangent drawn through that point of the curve at which the velocity is required. Let A B, fig. 67. be a curve in which a body moves, deſcribing equal areas in equal times (95, M), about the point C. Then the velocity at the points D and F will be inverſely as the perpendiculars C H, C I, let fall from C on the tangents D H, F I to the curve in the points D, F. For if the body move through the ſpaces D E, F G, in equal indefinitely ſmall portions of time, thoſe lines may be taken for portions of the tangents D H, F I, and the triangles D C E, F C G, will be equal (95, M). But the baſes D E, F G, of equal triangles, are inverſely as the perpendiculars C H, C I. And the velocities being as the baſes D E, F G, deſcribed in equal times, muſt alſo be in the ſame inverſe ratio of thoſe perpendiculars. Which was to be ſhewn. P

If a body, acted on by a centripetal force, directed to C (fig. 68.), be projected from U, in a direction at right angles to U C, but with a velocity too ſmall to cauſe it to revolve in a circle U A, it will fall within the circle, by the greater prevalence of the centripetal force. As it approaches the center, its velocity muſt increaſe (96, R), and ſo muſt likewiſe its tendency to recede from the center. Q center. If the centripetal force increaſes in the ſame or in a higher ratio than that tendency, the body will ſtill continue to approach, and at length fall into the center. But if the centripetal force increaſes in a leſs ratio, the increaſing velocity will cauſe the body to move in a courſe leſs and leſs inclined to the radius, till at length it becomes at right angles to it, and recedes again from the center, becauſe by the ſuppoſition, the velocity is too great for the body to move there in a circular orbit. In the receſs from the center the velocity muſt decreaſe (96, R), and a ſimilar curve be deſcribed by the body, till its courſe becomes again at right angles to the radius, and it is again cauſed to approach the center. And this alternation will continue for ever.

R The aſſertion reſpecting the ſimilarity or rather congruity of the curves between the apſides may be eaſily evinced from fig. 40. For if a body be ſuppoſed to move from H to C, and to be reflected back from C, in the direction and with the velocity C D, it is unneceſſary to ſhew that it would again deſcribe the ſame polygon C H. And the ſame holds good of curves (95, M). Now a body that arrives at its apſis muſt move with a velocity and direction which, with reſpect to the center, is equivalent to its being reflected back in the contrary direction, becauſe in either caſe it will begin to move with a given velocity in the tangent of the ſame circle.

It would carry us too far into the conſideration S
of the nature of thoſe curves that may be deſcribed
by bodies acted on by centripetal forces, if we
were to enquire minutely into the conſequences that
would follow from the ſuppoſition of various laws
of its increaſe or diminution, according to the diſtance.
Our purpoſe will be ſufficiently anſwered
by attending to the velocities of revolving bodies
in their apſides. Let a body (fig. 68.) be pro- T
jected from U towards A, in a direction at right
angles to CU, a line drawn from the point C,
to which let the centripetal force be ſuppoſed to be
directed. Suppoſe the velocity of projection to
be leſs than would be required to carry the body
in a circle at U, and the body will accede towards
the center, by paſſing through a curve UDF. If
the law of the centripetal force be ſuch (201, Q)
as to ſuffer the body to recede again, after coming
within a certain diſtance of the center, there will
be ſome point L, at which the body, previous to its
going off, will neither approach nor recede from
the center, but move in a direction at right angles
to the radius. This point is the lower apſis, and
its velocity will then be inverſely as the perpendicular
CL (201, P).

Let us ſuppoſe the centripetal force to be in- U
verſely as the cube of the diſtance from C. Then
(200, M) the velocities neceſſary to carry a body in
a circle at U or L will be inverſely as the diſtances
UC, LC. The actual velocities at U and L are in
the ſame ratio, and the velocity at U is known to
be

be too ſmall to carry it in a circle there. Conſequently, the actual velocity at L muſt be likewiſe too ſmall in the ſame ratio, and the body will continue to approach the center, after having paſſed the apſis. It will not, therefore, deſcribe a curve congruous with the curve deſcribed in its paſſage between the two apſides. But this laſt conſequence being contrary to what has been already proved (202, R) muſt be falſe, and ſo muſt likewiſe be the original ſuppoſition from which it was deduced.

V Conſequently it is not true, that a body projected with a velocity too ſmall to keep it in a circle, and acted on by a centripetal force inverſely as the cubes of the diſtances, can ever arrive at the lower W apſis. It muſt therefore continually approach the center, and at length fall into it.

X If the body be ſuppoſed to be originally projected from L, the lower apſis, with a velocity too great for the centripetal force, according to the ſame law, to retain it in a circle, it may be ſhewn by ſimilar argumentation that it would never arrive at the higher apſis, but would continually recede from the center.

Y Thus it appears, that the inverted ratio of the cubes of the diſtances is the law of centripetal force that limits the revolutions of bodies in curves that admit of alternate approach and receſs from the center. For if, according to this law, a body, after once beginning to approach the center or to recede from it, cannot but continue that approach or receſs, it muſt be much more ſtrongly urged in the ſame manner by a centripetal force that follows the inverted ratio of ſome higher power of the diſtance. And again, if the law of the force follows ſome inverted ratio leſs than that of the cube of the diſtance, the velocities required to retain bodies in circular orbits will be leſs than after the inverted ratio of the diſtance as that law would require (200, L, M). Whence it follows, that ſince the velocities in deſcending from the upper apſis increaſe faſter than the diſtances decreaſe, the perpendicular CH, fig. 67. being leſs than the diſtance CD (201, P), the motion of the body will be directed leſs and leſs towards the center, till it becomes at right angles to the radius, the body being then in the lower apſis. After which it muſt aſcend through a curve ſimilar and equal to that it before deſcribed in paſſing between the apſides (202, R).

Z Hence it is ſeen, that when the centripetal force increaſes in approaching the center in a leſs ratio than the inverſe ratio of the cubes of the diſtances, the law of its increaſe may be found from the quantity of angular motion employed in paſſing from one apſis to the other. For the diſtance between A the upper and lower apſis will be greater the nearer the law of the centripetal force approaches to that ratio, becauſe the body muſt run through a greater ſpace before the tendency to recede from the center, ariſing from the velocity and direction, can be equal to the centripetal force.

B If a body revolves in an elliptical orbit, deſcribing equal areas in equal times about one of the foci, ſoci, the apſides will be at the two extremities of the tranſverſe diameter, or 180° of angular motion apart, the centripetal force will be directed to that focus (95, N), and its intenſity will be inverſely as the ſquare of the diſtance *.

C If a body revolves between two apſides, and the centripetal force be inverſely as ſome power of the diſtance, greater than the ſquare and leſs than the cube, the diſtance between the apſides will be more than 180°. But if the centripetal force be inverſely as ſome power of the diſtance leſs than the ſquare, the diſtance between the apſides will be leſs than 180° (205, A). In theſe caſes the orbit may be conſidered as an ellipſis whoſe tranſverſe diameter, or line of the apſides, is not ſtationary, but revolves on the focus to which the force

D is directed. The apſides may therefore be ſaid to revolve in conſequentia, or with the moving body, when the force in approaching the center is greater than after the inverſe ratio of the ſquare of the diſtance; or to revolve in antecedentia when the

E force is leſs than after that ratio. And the quieſcence of the apſides will be a proof, that the centripetal force is accurately in the inverſe ratio of the ſquare of the diſtance.

F The periodical time of a revolution about the focus in a quieſcent ellipſis is equal to that which would be employed in deſcribing a circle whoſe radius is half the tranſverſe diameter of the ellipſis †.

---

* Principia, I. §§. 3. 9.

† Principia, I. 15.

Therefore, if the ſquares of the periodical times G
of bodies revolving in ellipſes be directly as the
cubes of the mean diſtances (200, O) the centripetal forces will be inverſely as the diſtances.

CHAP. II.

THE UNIVERSALITY OF GRAVITATION DEDUCED FROM ITS EFFECTS.

THE planetary bodies being in motion would
(21, P) continue to move for ever in right
lines, unleſs compelled to change their ſtate by
forces impreſſed. But they move in curve lines H
(121, K), and conſequently muſt be acted on by
forces that continually deflect their courſes out of
the right lined direction.

Every primary planet moves with ſuch a velocity I
and direction, that a line joining the centers of the
planet and the Sun deſcribes equal areas in equal
times (121, K). Whence it follows, that the centripetal forces which retain theſe planets in their
orbits are (95, N) directed to the Sun's center.

The periodical times of the primary planets are
ſuch, that their ſquares are directly in proportion
to the cubes of their mean diſtances from the Sun.
Their orbits are (121, K) elliptical, and their apſides quieſcent. From theſe phenomena it is proved K
(207, G. 205, B. 206, E), that the centripetal forces
are inverſely as the ſquares of the diſtances from
the Sun.

Every ſecondary planet moves with ſuch a velocity and direction, that a line joining the center of the ſecondary with that and its primary, deſcribes equal areas in equal times. The centripetal forces retaining theſe bodies in their orbits conſequently are (95, N) directed to the centers of their reſpective primaries. L

The periodical times of Saturn's moons are ſuch, that their ſquares are directly in proportion to the cubes of their diſtances. And, therefore, the centripetal forces are inverſely as the ſquares of the diſtances (200, O). M

N The ſame phenomenon in Jupiter's moons ſhew, that their centripetal forces follow the ſame law.

O The orbits of Saturn's and Jupiter's moons are here taken to be circular. For obſervation has not yet eſtabliſhed the eccentricity of any of theſe orbits, except that of Jupiter's fourth ſatellite.

The Moon is carried about the Earth with ſuch a velocity and direction, that a line joining its center and that of the Earth, deſcribes equal areas in equal times (143, F). It is therefore retained in its orbit by a force directed to the Earth's center. P

Q The Moon's orbit is elliptical, and its apſides quieſcent. Its centripetal force is therefore inverſely as the ſquare of its diſtance (205, B. 206, E).

Every comet moves with ſuch a velocity and direction, that a line joining the centers of the comet and Sun deſcribes equal areas in equal

times (160, V). The centripetal forces retaining R
the comets in their orbits is (95, N) therefore directed
to the Sun.

All the comets deſcribe either ellipſes or parabolas
(160, U, V). Thoſe whoſe return has been obſerved
have their apſides quieſcent. Whence it S
follows * (205, B. 206, E), that the centripetal
forces are inverſely as the ſquares of the diſtances
from the Sun.

T It is not to be underſtood that the planetary
phenomena are in abſolute ſtrictneſs as given in this
place. But the irregularities are very ſmall, and
it will hereafter be ſeen, that they are of ſuch a
nature as to give additional force to the deductions
here made.

U The force that retains the Moon in its orbit is
the ſame with that which cauſes bodies near the
Earth's ſurface to be heavy, and is called Gravity.

V To prove this important truth, let us take 57′
for the Moon's horizontal parallax at its mean
diſtance, and that diſtance will, by plane trigonometry,
be found to be 60.314 ſemidiameters of the
Earth. The periodical time of the Moon is 27 d.
7 h. 43 m. (142, A), or 39343 minutes, which is
the ſame period as would obtain if its orbit were
a circle (206, F) whoſe radius is equal to the mean
diſtance, and the centripetal force remained unaltered.
To come at the effects of this force more
readily, it will be convenient to attend to this circular

---

* For the parabola, ſee Principia, I. 13.

cular revolution. In one minute of time the Moon in this orbit would paſs through the $\frac{1}{34393}$ part, or an arc of 32.941 ſeconds of meaſure. The mean length of a degree on the Earth's meridian is 342516 Paris feet, which number multiplied by 60.314, will give 20658510 Paris feet for the length of a degree in the ſuppoſed circular orbit. Whence the arc of 32.941 ſeconds paſſed through in a minute may be found in feet, as alſo its verſed ſine. The verſed ſine being the ſpace through which the Moon muſt fall beneath the tangent in the time of one minute, will be the effect of, and will meaſure the centripetal force (35, E). This ſpace or verſed ſine is 15.0944 Paris feet. Now, becauſe the Moon's centripetal force is inverſely as the ſquare of the diſtance from the Earth's center (208, Q), we may find what its effect would be at the Earth's ſurface by ſaying, As the ſquare of 60.314, the Moon's diſtance from the Earth's center; Is to the ſquare of 1, the diſtance of the ſurface of the Earth from its center: So is the meaſure (35, E) of the centripetal force at the Moon 15.0944 feet; To the meaſure or effect of the ſame force at the Earth's ſurface, or 54910 feet.

W To find whether this ſpace agrees with the fall of bodies at the Earth's ſurface by gravity, we muſt reduce the time to one ſecond; becauſe we have no means of directly meaſuring the actual fall of bodies during ſo long a time as one minute. Now, though the centripetal force of the Moon increaſes in approaching the Earth's center, yet theſe meaſures of that force are ſtrictly accurate, becauſe conſidered as they obtain in circular orbits where the diſtance of the body is not diminiſhed by its fall: but the difference would in the preſent caſe be abſolutely inſenſible in ſo ſhort times as a ſecond or a minute, even if we ſuppoſed the falling body to be moved in a right line directed to the center. The ſpaces deſcribed by falling will conſequently be deſcribed by an uniformly accelerated motion, and will be (29, G) as the ſquares of the times. Therefore as the ſquare of 60 ſeconds; Is to the ſquare of 1 ſecond: So is 54910 feet; To 15.2528 feet that bodies would fall through at the Earth's ſurface in a ſecond by the action of the centripetal force that retains the Moon in its orbit.

But bodies fall through 15.084 Paris feet in a X ſecond, by the action of gravity. The fall of bodies near the Earth's ſurface and of the Moon are effects of the ſame kind, and therefore, by the ſecond rule of philoſophizing, are produced by the ſame cauſe. That is to ſay, the Moon is retained in its orbit by gravity.

Moreover, ſince it is eſtabliſhed from the Moon's Y revolution in its orbit that a centripetal force exiſts and acts in the direction towards the center of the Earth, and no good reaſon can be given againſt its action that would take place, according to its law, on bodies any where ſituated, it muſt follow, that bodies fall near the Earth's ſurface, either by this force alone or by this force in conjunction with ſome other. But this latter conſequence cannot be admitted, becauſe bodies would then fall with greater velocity than the Moon at the ſame diſtance; whereas it has been juſt ſhewn, that their velocities are ſomewhat leſs.

Z And even this difference between the velocities of the Moon and of ſmaller bodies affords an additional proof, that the ſame force of gravity is concerned in both. For the force would have proved accurately the ſame, as far as obſervation can meaſure its effects, if proper allowance, according to the laws of gravity known from its effects on heavy bodies, had been made in the computation for the maſs of the Moon, and the Sun's action on the Earth and Moon.

A The revolutions of the ſatellites of Jupiter and Saturn, and alſo thoſe of the primary planets and of the comets, are phenomena of the ſame kind as the Moon's revolution about the Earth, and are therefore (6, II.) to be attributed to the ſame cauſes, namely, to an original or projectile motion, compounded with a motion produced by gravity.

B Action and re-action (22, R) being equal, it follows alſo, becauſe the ſecondary planets gravitate towards their primaries, and the primary planets together with their ſatellites, as likewiſe the comets, gravitate towards the Sun, that the primary planets muſt likewiſe gravitate towards their ſecondaries, and the Sun towards the whole
C ſyſtem. That is to ſay, gravitation is univerſal, or a property of all bodies whatſoever.

The mutual action of two bodies (22, R. 95, O) D that gravitate to each other is the cauſe that if they fall, both will approach the common center of gravity with equal quantities of motion, or if they have at the ſame time a projectile velocity, both will abſolutely revolve about that center. If one of the two bodies exceed the other indefinitely in maſs, its velocity (19, L) of approach will be indefinitely leſs than that of the other in the former caſe, or the radius of its orbit will be indefinitely leſs than that of the other in the latter caſe. Whence the whole relative velocity of ap- E proach may be taken for the abſolute velocity produced in the leſs body; or the orbit deſcribed by the leſs body about the greater may be taken for that which it deſcribes about the common center of gravity.

The fall of bodies near the Earth's ſurface may F be regarded as their abſolute motion. For the magnitude of the Earth is ſo great with reſpect to the bodies with which art can make experiments, that its velocity is incomparably ſmaller than the differences which the imperfections of the ſenſes muſt cauſe in all obſervations. We may G likewiſe in this place conſider the motions of the planets and comets about the Sun as abſolute, though the magnitude of that luminary is not ſo exceſſive as to render thoſe of the planets inconſiderable. The ſame is to be underſtood of the H ſatellites of Jupiter and Saturn.

All bodies fall near the Earth's ſurface with I equal velocities (26, A, B). The planets and comets being accelerated towards the Sun by powers which are (207, K. 209, S) inverſely as the ſquares of the diſtances, would conſequently be equally accelerated at equal diſtances. The ſame is true of the ſatellites with reſpect to the Sun, becauſe they revolve together with their primaries in the annual orbits, and alſo with reſpect to their primaries (208, M, N).

K From theſe equal accelerations it follows, that the force of gravity which urges minute bodies towards any other larger body, is in proportion to the maſs of the body urged.

L This law of gravity being eſtabliſhed from obſervations in caſes where the velocity of the ſmaller body can be taken without ſenſible error, we may again reſume the conſideration of the re-action of the ſmaller. Let us call the larger L, and the ſmaller S. Then, becauſe S is urged towards L, by a force which is as the magnitude of S, L will be urged (22, R) towards S, with the ſame force. That is, if S becomes larger, it will attract L the more ſtrongly in proportion. Now, S may be imagined to become larger, ſo as even to exceed L in any ratio whatever, and the increaſe of at-

M traction will ſtill obtain. Therefore, a given body not only attracts another, in proportion to the maſs of this laſt, but alſo in proportion to its own

N maſs. That is to ſay, the force of gravitation exerted between two bodies is in the compound ratio of their maſſes.

O The abſolute force of gravity being in the compound ratio of the maſſes of the two bodies, or as the maſſes multiplied into each other, will be meaſured by the quantity of motion produced in either body (21, Q. 19, L) in a given time, or by the maſs of the body multiplied into its velocity; that is, the numeral product of the maſſes will be equal to the numeral product of the maſs of one of the bodies into its velocity. Now, if each of theſe equal products be divided by the maſs of the body, whoſe velocity is conſidered, the two remainders will be equal, namely, the maſs of the one body will be equal to the velocity of the other. Or, more P clearly (becauſe we have uſed number, in order to avoid the compariſon of ratios, which is leſs generally underſtood) the maſs of one body will be as the velocity or acceleration of the other. Whence Q it follows, that if the velocities produced in bodies by gravitation be known, the proportional maſſes of the bodies towards which they are urged will be alſo known.

From this conſequence the maſs of any large pla- R net may be known from the velocity of deſcent it produces in bodies indefinitely ſmaller than itſelf. For the relative velocities of ſuch ſmall bodies (213, E), with reſpect to the larger, may be taken for their abſolute velocities. Now, the velocities of the planets S towards the Sun, of the ſatellites of Jupiter and Saturn towards their reſpective primaries (213, G), and of projectiles near the Earth's ſurface, being reduced to equal diſtances of the attracting bodies, as was done in comparing the Moon's gravity with that of terreſtrial bodies (209, V, W, X) or otherwiſe, are, and conſequently likewiſe their maſſes are, (nearly) as the numbers 1, $\frac{1}{1067}$, $\frac{1}{3021}$, and $\frac{1}{169282}$. Hence alſo the denſities may be found, becauſe their bulks T are known from obſervation. Thus, the denſities, or ſpecific gravities, are in four bodies juſt mentioned, 100, $94\frac{1}{2}$, 36, and 400. Theſe numbers may be familiarized to the imagination, by obſerving, that if the mean denſity of the Earth be ſuppoſed to anſwer to that of common green glaſs, the Sun's denſity would be equal to that of dry pear-tree, Jupiter's to cedar, and Saturn's to cork.

U Our knowledge of the remote parts of the planetary ſyſtem is too imperfect to admit of many re- V marks on the facts we can obſerve and deduce. It is, however, worthy of notice, that the immenſely large planets, Jupiter, Saturn, and the Georgium Sidus, would have occaſioned great irregularities by their attractions on the other bodies of the ſyſtem; if, inſtead of being placed at the great diſtances they are from the common center of gravity, and from each other, they had occupied the places of the ſmall planets, Mars, the Earth, Venus or Mer- W cury. And if the celeſtial ſpaces be not abſolutely vacuous, but poſſeſſed by ſome very rare matter, (175, A) the reſiſtance ſuch matter muſt afford in the courſe of ages to the motions of the planets, will be brought nearly to equality in its effects, if the planets which move ſwifter are at the ſame time more denſe. Laſtly, becauſe it is obſerved in the conſtitution of terreſtrial bodies, that the denſer require, in many inſtances, a higher degree of heat to produce given changes in them, it has been conjectured,

jectured, not without ſome degree of probability, that the planets nearer the Sun are for this purpoſe formed of denſer materials adapted to their ſituation.

From the univerſality of gravitation (212, c) x it is deduced, that the fixed ſtars are either falling towards the common center of gravity of the univerſe, or are made to deſcribe immenſe orbits which ultimately reſpect that center. The life of man, aſſiſted by every traditional record for thouſands of years, ſeems too ſhort to aſcertain the reſult of this ſublime enquiry. It will be a grand acquiſition, if the repeated obſervations of ſeveral centuries to come ſhould determine the proper motions of the vaſt number of ſuns that compoſe the nebulæ, (194, z) of which our whole planetary ſyſtem, with all its comets, forms ſo inconſiderable a part.

CHAP.

CHAP. III.

OF THE IRREGULARITIES ARISING FROM THE MUTUAL GRAVITATION OF THE PLANETS.

Y IF the Sun were at reſt, and the planets did not mutually gravitate towards each other, they would deſcribe ellipſes, having the Sun in the common focus. But ſince they mutually act on the Sun, and on each other (212, c) it muſt follow that the Sun is perpetually moved about the center of gravity of all the planets, which center is the common focus of their orbits. This center, by reaſon of the Sun's very great bulk, can, in no ſituation, exceed the diſtance of its ſemidiameter from its ſurface. Z Some ſmall irregularities ariſe from theſe mutual actions, but much leſs than would enſue if the Sun were at reſt, or not ſubject to the re-action of the other planets. The irregularities in the motions of the primary planets are ſcarcely conſiderable enough to come under obſervation in the courſe of many revolutions: thoſe of the Moon, on account of its nearneſs to us, and from other cauſes, have ever been ſufficiently great, to embarraſs the aſtronomical world. It will therefore be ſufficient to explain the latter, and apply the explanation to the former, which are effects of the ſame kind.

A If the actions of the Sun upon the Earth and Moon were equal upon each, according to their maſſes, and tended to produce motions in parallel directions, their relative motions would be the ſame as if no ſuch forces acted upon them (79, w). But theſe forces vary, both in quantity and direction, according to the various relative ſituations of the Earth and Moon.

B Let the point s (fig. 66.) repreſent the Sun, E the Earth, and A D B C the orbit of the Moon. Then, if the Moon be at the quadrature A, the diſtances E S and A S of the Earth and Moon from the Sun, being equal, their gravities towards S will alſo be equal, and may be repreſented by thoſe lines E S and A S. Draw the line A L parallel and equal to E S, and join L S, which will be parallel to AE. The force A S may be reſolved (23, T) into the two forces A L and A E; of which A L, by reaſon of its paralleliſm and equality to E S (79, w) will not diſturb their relative motions or ſituation: but the force A E, conſpiring with that of gravity, will cauſe the Moon to fall farther below the tangent of its orbit than it would have done if no ſuch diſturbing force had exiſted. Therefore, at or near the quadratures, the C Moon's gravity towards the Earth is increaſed more than according to the regular courſe, and its orbit is rendered more curve.

D When the Moon is at the conjunction C, the diſtances E S and C S not being equal, the Moon's gravitation towards the Sun exceeds that of the Earth in the ſame proportion as the ſquare of E S exceeds the ſquare of C S. And becauſe the exceſs acts contrary to the direction of the Moon's gravity towards the Earth, it diminiſhes the effect thereof, and

cauſes the Moon to fall leſs below the tangent of its
orbit than it would if no ſuch diſturbing force
exiſted. A like, and very nearly equal, effect fol-
lows, when the Moon is at the oppoſition D, by the
Earth's gravitation towards the Sun being greater
than that of the Moon; whence their mutual gra-
E vity is diminiſhed as in the former caſe. Therefore,
at or near the conjunction or oppoſition, the Moon's
gravity is diminiſhed, and its orbit is rendered leſs
curve.

F It is found, that the force added to the Moon's
gravity at the quadratures, is to the gravity with
which it would revolve about the Earth in a circle
at its preſent mean diſtance, if the Sun had no effect
on its motion, as 1 to 190; and that the force ſub-
ducted from its gravity at the conjunction or oppo-
G ſition is about double this quantity. The influence
of the Sun, then, on the whole, increaſes the Moon's
diſtance from the Earth, and augments its periodi-
cal time; and ſince this influence is moſt conſi-
derable when the Earth is neareſt the Sun, or in
its perihelium, its periodical time muſt then be
the greateſt, as appears likewiſe from obſervation
(144, K).

H To ſhew the effect of the Sun in diſturbing the
Moon's motion at any ſituation between the con-
junction and one of the quadratures, ſuppoſe at M
(fig. 66.) let ES repreſent the Earth's gravity to-
wards the Sun; draw the line MS, which continue
towards G; from M ſet off MG, ſo that MG may be
to ES as the ſquare of the Earth's diſtance ES is to

the ſquare of the Moon's diſtance M S; and M G will repreſent the Moon's gravity towards the Sun. From M draw M F parallel, and equal to E S; join F G, and draw M H parallel, and equal to F G. The force M G may be reſolved into M F and M H; of which M F, by reaſon of its parallelism and equality to E S, will not diſturb the relative motions or ſituations of the Moon and Earth; M H then is the diſturbing force. Draw the tangent M K to the Moon's orbit, and continue the radius E M towards I; draw H I parallel to K M, and interſecting M I in I, and complete the parallelogram by drawing H K parallel to I M, and interſecting M K in K. The force M H may be reſolved into M I and M K; of which M I affects the gravity, and M K the velocity of the Moon. When the force M H coincides with the tangent; that is, when the Moon is 35° 16′ diſtant from the quadrature, the force M I, which affects the gravity, vaniſhes; and when the force M H coincides with the radius, that is, when the Moon is either in the conjunction or quadrature, the force M K vaniſhes. Between the quadrature and the diſtance of 35° 16′ from it, the line or force M H falls within the tangent, and conſequently the force M I is directed towards E, and the Moon's gravity is increaſed: but, at any greater diſtance from the quadrature, the line M H falls without the tangent, and the force M I is directed from E, the Moon's gravity being diminiſhed. It is evident that the force M K is always directed to ſome point in the line which paſſes through the centers of the Sun and Earth;

therefore it will accelerate the Moon's motion, while it is approaching towards that line, or the conjunction, and ſimilarly retard it as it recedes from it, or approaches towards the quadrature, by conſpiring with the motion in one caſe, and ſubducting from it in the other.

K As the Moon's gravity towards the Sun at the conjunction is diminiſhed by a quantity which is as the difference of the ſquares of their diſtances; and as this difference, on account of the very great diſtance of the Sun, is nearly the ſame when the Moon is at the oppoſition, the mutual tendency to ſeparate, or diminution of gravity, will be very nearly the ſame. Whence it eaſily follows, that all the irregularities which have been explained as happening between the quadratures and conjunction muſt in like circumſtances take place between the quadratures and the oppoſition.

L If the Moon revolved about the Earth in a circular orbit, the Sun's diſturbing influence being ſuppoſed not to act, then this influence being ſuppoſed to act would convert the orbit into an ellipſis. For the increaſe of gravity renders it more curve at the quadratures, by cauſing the Moon to fall further below the tangent; and the diminution of gravity, as well as the increaſed velocity, renders the orbit leſs curve at the conjunction and oppoſition, by cauſing the Moon to fall leſs below the tangent in a given time. Therefore an ellipſis would be deſcribed, whoſe leſs or more convex parts would be at the quadratures, and whoſe longeſt diameter would paſs through them. Conſequently the Moon would be fartheſt from the Earth at the quadratures, and neareſt at the conjunction and oppoſition. Neither is it ſtrange that the Moon ſhould approach or come nearer to the Earth at the time when its gravity is the leaſt, ſince that approach is not the immediate conſequence of the decreaſe of gravity, but of the curvity of its orbit near the quadratures; and in like manner, its receſs from the Earth at the quadratures does not ariſe immediately from its diminiſhed gravity, but from the velocity and direction acquired at the conjunction or oppoſition.

But as the Moon's orbit is, independent of the Sun's action, an ellipſis, theſe effects take place only as far as circumſtances permit. M

The Moon's gravity towards the Earth being thus ſubject to a continual change in its ratio, its orbit is of no conſtant form. The law of its gravity being nearly in the inverſe proportion of the ſquares of the diſtances, its orbit is nearly a quieſcent ellipſis (206.); but the deviations from this law occaſions its apſides to move direct or retrograde, according as thoſe deviations are in defect or exceſs (206, c). Aſtronomers, to reduce the motion of the apſides to computation, ſuppoſe the revolving body to move in an ellipſis, whoſe tranſverſe diameter or line of the apſides revolves at the ſame time about the focus of the orbit. When the Moon is in the conjunction or oppoſition, the Sun ſubducts from its gravity (220, E), and that the more the greater its diſtance is from the Earth, ſo that N that its gravity follows a greater proportion than the inverted ratio of the ſquare of the diſtance, and conſequently the apſides of its orbit muſt then move in conſequentia, or direct (206, D). In the quadratures the Sun adds to the Moon's gravity, (219, O) and that the more the greater its diſtance from the Earth, ſo that its gravity follows a leſs proportion than the inverted ratio of the ſquare of her diſtance, and conſequently the apſides of its orbit muſt then move in antecedentia, or retrograde (206, E). O But becauſe the action of the Sun ſubducts more from the Moon's gravity in the conjunction and oppoſition than it adds to it in the quadratures (220, F) the direct motion exceeds the retrograde, and at the end of each revolution the apſides are found to be advanced according to the order of the ſigns.

P If the plane of the Moon's orbit coincided with that of the ecliptic, theſe would be the only irregularities ariſing from the Sun's action; but becauſe it is inclined to the plane of the ecliptic in an angle of about five degrees, the whole diſturbing force does not act upon the Moon's motion in its orbit, a ſmall part of the force being employed to draw it out of the plane of the orbit into that of the ecliptic.

Q Of the forces M K and M I, fig. 66. which diſturb the Moon's motion, M I being always in the direction of the radius, can have no effect in drawing it out of the plane of its orbit. And if the force M K really coincided with the tangent, as we, neglecting the ſmall deviation ariſing from the obliquity of the Moon's orbit have hitherto ſuppoſed, it is evident that its only effect would be that of accelerating or retarding the Moon's motion, without affecting the plane of its orbit. But becauſe that force is always directed to ſome point in the line which paſſes through the centers of the Sun and Earth (221, 1) it is evident that it can coincide with the tangent only when that line is in the plane of the Moon's orbit; that is to ſay, when the nodes are in the conjunction and oppoſition. At all other times the force MK muſt decline to the northward or ſouthward of the tangent, and compounding itſelf with the Moon's motion, will not only accelerate or retard it, according to the circumſtances before explained, but will likewiſe alter its direction, deflecting it towards that ſide of the orbit on which the point, the force MK, tends to, is ſituated. This deflection cauſes the Moon to arrive at the ecliptic either ſooner or later than it would otherwiſe have done; or, in other words, it occaſions the interſection of its orbit with the ecliptic to happen in a point of the ecliptic, either nearer to, or further from, the Moon, than that in which it would have happened if ſuch deflection had not taken place.

To illuſtrate this, let the elliptical projection RCOQN (fig. 67.) repreſent a circle in the plane of the ecliptic, MOPN the Moon's orbit, interſecting the ecliptic in the nodes N and O. Suppoſe the Moon to be in the northern part of its orbit at M, and moving towards the node O; the diſturbing force MK, M K, which tends towards a point in the line S E to the ſouthward of the tangent M T, will be compounded with the tangental force, and will cauſe the Moon to deſcribe the arc M m, to which M R is tangent, inſtead of the arc M O; whence the node O is ſaid to be moved to m. In this manner the motion of the nodes may be explained for any other ſituation.

S This motion evidently depends on a twofold cir-
cumſtance, namely, the quantity and direction of
the force M K. If the force M K be increaſed, its
direction remaining the ſame, it will deflect the
curve of the Moon's path from its orbit in a greater
degree; and on the other hand, if its direction be
altered, ſo as to approach nearer to a right angle
with the tangent, it will cauſe a greater deflection,
T though its quantity remain the ſame. When the
Moon is in the quadratures, the force M K vaniſhes,
(220, H) conſequently the nodes are then ſta-
tionary. When the Moon is at the octant, or
forty-five degrees from the quadrature, the force
M K is greateſt of all, and therefore the motion
of the nodes is then moſt conſiderable, as far as it
U depends on the quantity of M K. But the direc-
tion of this force in like circumſtances depends on
the ſituation of the line of the nodes. If the line
of the nodes coincides with the line paſſing through
the centers of the Sun and Earth, the force M K
coincides with the tangent of the Moon's orbit,
and the nodes are ſtationary. And the farther the
node is removed from that line, the farther is that

line removed from the plane of the Moon's orbit; till the line of the nodes is in the quadratures, at which time the line paſſing through the centers of the Sun and Earth, makes an angle with the plane of the Moon's orbit equal to its whole inclination, or five degrees: conſequently the angle formed between M K and the tangent in like circumſtances is then greateſt, M K being directed to a point in a line which is further from the plane of the Moon's orbit than at any other time, and of courſe the motion of the nodes is then moſt conſiderable.

To determine the quantity and direction of the motion of the nodes, ſuppoſe the Moon in the quarter preceding the conjunction, and the node towards which it is moving to be between it and the conjunction: in this caſe its motion is directed to a point in the ecliptic, which is leſs diſtant than the point towards which the force M K is directed: the force M K then, compounding with the Moon's motion, cauſes it to be directed to a point more diſtant than it would otherwiſe have been; that is to ſay, the node, towards which the Moon moves, is moved towards the conjunction. When the Moon has paſſed the node, its courſe is directed to the other node, which is a point in the ecliptic more diſtant than the point to which M K is directed, and therefore M K, compounding with its motion, cauſes it to be directed to a point leſs diſtant than it would otherwiſe have been; ſo that in this caſe likewiſe, the enſuing node is moved towards the conjunction. After the Moon has paſſed the conjunction, the force MK ſtill continues to deflect its courſe towards the ecliptic, and conſequently the motion of the node is the ſame way till its arrival at the quadrature. Suppoſe again, the Moon to be at the conjunction, and the node towards which it is moving to be between it and the quadrature. In this caſe the force MK compounding with the Moon's motion, cauſes it to move towards a point in the ecliptic leſs diſtant than it would otherwiſe have done, ſo that the enſuing node is brought towards the conjunction. When the Moon has paſſed the node, the force MK ſtill continuing to deflect its courſe towards the ſame ſide of its orbit, produces a contrary effect, namely, as it before occaſioned it to converge to the ecliptic, ſo it now cauſes it to diverge from it, and its motion in conſequence tends continually to a point in the ecliptic more diſtant than it would otherwiſe have done: the enſuing node in this inſtance being alſo brought towards the conjunction.

w As the diſturbing forces are very nearly the ſame in the half of the Moon's orbit, (222, K) which is fartheſt from the Sun, this laſt paragraph is true, when it moves in that part of its orbit, if the word oppoſition be every where inſerted inſtead of the word conjunction.

x Whence it is eaſy to deduce this general rule, that when the Moon is in the part of its orbit neareſt the Sun, the node towards which it is moving is made to move towards the conjunction: and when it is in the part of its orbit fartheſt from the Sun, the node towards which it is moving is made to move towards the oppoſition.

Suppoſe the Moon at Q, (fig. 68.) or the quadrature preceding the conjunction, then the enſuing node, if at 90° diſtance, or at the conjunction C, will be ſtationary (226, U) but if it be at a greater or leſs diſtance, it will be brought towards C (228, X). Thus, if the nodes be in the poſition M N, the enſuing node M, being at a leſs diſtance from Q than 90°, will move towards C, or direct, while the Moon moves through the arc Q M; after which N becomes the enſuing node, and likewiſe moves towards the conjunction C, or retrograde during the Moon's motion through the arc M R. And becauſe the arc M R exceeds Q M, the retrograde motion exceeds the direct. Again, if the nodes be in the poſition n m, the enſuing node n being at a greater diſtance from Q than 90°, will move towards C, or retrograde, during the Moon's motion through the arc Q n; after which the node m becomes the enſuing node, and likewiſe moves towards the conjunction C, or direct, during the Moon's motion through the arc n R. And becauſe the arc Q n exceeds N R, the retrograde motion here alſo exceeds the direct. If the nodes be in the quadratures Q R, the enſuing node R removes towards C, or retrograde, during the Moon's motion through the arc Q R, or almoſt the whole ſemi-orbit. The ſame may be ſhewn in the other half of the orbit R O Q with reſpect to the oppoſition O; and therefore, in every revolution of the Moon, the retrograde motion of the nodes ex- Y

Z ceeds the direct; and, on the whole, the nodes are carried round, contrary of the order of the ſigns.

A The line of the conjunction is by the Earth's annual motion brought into every poſſible ſituation with reſpect to the nodes in the courſe of a year, independant of their own proper motion; which laſt occaſions the change of ſituation to be performed in about nineteen days leſs.

B The inclination of the Moon's orbit being the angle which its courſe makes with the plane of the ecliptic, it is evident from what has been ſaid, that this angle is almoſt continually changing. Suppoſe the line of the nodes, by its retrograde motion, to leave the conjunction C, fig. 69. and become in the ſecond and fourth quarters as in the poſition M N, and the Moon to move from the node M to the node N; then, becauſe the enſuing node N moves (228, X) towards the conjunction C, while the Moon is in the nearer half of its orbit, the Moon's courſe muſt be continually more and more inflected towards the ecliptic, till its arrival at R. This inflection in the firſt 90°, or M A from M, prevents its diverging ſo much from the ecliptic as it would otherwiſe have done; that is to ſay, it diminiſhes the angle of the Moon's inclination. From A to R its courſe begins to converge towards the ecliptic, and this convergence is increaſed by the inflection which in the preceding 90° prevented its divergence: in the arc A R then the inclination is increaſed. During the Moon's motion from R to N, the node is moved towards the oppoſition

poſition O, and conſequently the angle of its courſe to N is rendered leſs than it would have been if the node has not moved; or, in other words, the inclination is diminiſhed. And becauſe the arc M A added to the arc R N is greater than the arc A R, the inclination at the ſubſequent node is leſs than at the precedent node; and the ſame may be ſhewn in the other half revolution N Q M. Therefore, C
while the nodes are moving from the conjunction and oppoſition to the quadratures, the inclination of the Moon's orbit, on the whole, diminiſhes in every revolution till they arrive in the quadratures, at which time it is leaſt of all. When the line of D
the nodes has paſſed the quadratures, and is in the firſt and third quarters, as in the poſition m n, it is eaſily ſhewn by the ſame kind of argument, that the inclination is increaſed while the Moon paſſes from m to Q; then diminiſhes for the remainder of the firſt 90° or Q a, and is afterwards increaſed for the other 90° or a n: and the ſame may be proved for the other half revolution n R m. Conſequent- E
ly, while the nodes are moving from the quadratures to the conjunction and oppoſition, the inclination is increaſed by the ſame degrees as it before was diminiſhed, till they arrive at the conjunction and oppoſition, at which time it returns to its firſt quantity, being then greateſt of all.

The line of the nodes in the courſe of one en- F
tire revolution, with reſpect to the Sun, is twice in the quadratures and twice in the conjunction and oppoſition. Therefore, the inclination of the

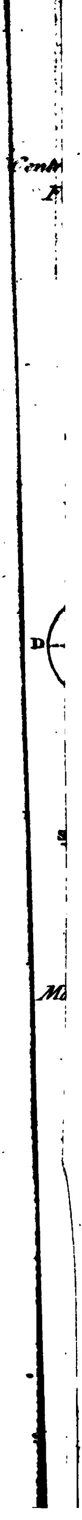
Moon's orbit to the ecliptic is diminiſhed and increaſed by turns, twice in every revolution of the nodes.

G All the irregularities of the Moon's motion are a little greater when in the half of its orbit neareſt the Sun, than when it is in the other half; the chief reaſon of which is, that the difference between the ſquares of the Moon's and Earth's diſtances from the Sun is greater, in proportion to the ſquares themſelves, in the former than in the latter caſe at equal elongations from the quadrature, and conſequently the diſturbing forces muſt be more conſiderable.

H Although the Moon in reality revolves about the common center of gravity between her and the Earth, and not about the Earth itſelf, and conſequently their motions and irregularities are ſimilar, and not confined to the Moon alone; yet it may be eaſily conceived, that the concluſions are not affected in any degree that may be here regarded, when, for the ſake of conciſeneſs, we ſuppoſe one of the two bodies to be quieſcent, and the other to revolve about it.

I Irregularities of the ſame kind take place among the primary planets by their mutual actions on each other, but the quantities are not conſiderable. Hence the apſides of the planets are found to move (224, 0) in conſequentia, but ſo very ſlowly, that ſome have doubted whether they move at all.

K The motions of the aphelia of Saturn, Jupiter, Mars, the Earth, Venus, and Mercury, as deduced from the compariſon



UNIV. OF MICH.

comparifon of diſtant obſervations, are reſpectively, 2° 30′, 1° 43′ 20″, 1° 51′ 40″, 1° 49′ 10″, 4° 10′, 1° 57′ 40″, in a century. The actions L
of the inferior planets on each other are very minute, on account of the ſmallneſs of their bulks; but thoſe of Jupiter and Saturn are not altogether inſenſible. When Jupiter is between the Sun and M
Saturn, its whole attraction acts upon Saturn, and increaſes the gravity of that planet towards the Sun. This is found, by comparing the reſpective maſſes of Jupiter and the Sun, and the reſpective ſquares of their diſtances from Saturn, to be equal to $\frac{1}{321}$ of the Sun's action upon Saturn. Saturn, N
on the other hand, at the conjunction, acts upon Jupiter and the Sun in the ſame direction, and therefore diſturbs their relative poſition only ſo far as its actions on each are not equal. The difference of theſe actions is found by the ſame principles to be $\frac{1}{1923}$ of Jupiter's whole gravity.

CHAP.

CHAP. IV.

OF THE FIGURES OF THE PLANETS; THE PRECESSION OF THE EQUINOXES, AND THE NUTATION OF THE EARTH'S AXIS.

O A Maſs of fluid matter will, by its gravity, form itſelf into a ſphere. For if the whole maſs be conceived to be divided into a number of ſimilar pyramids or columns, terminating in the center of gravity, and one of theſe columns be longer or higher than the reſt, its projecting part will ſpread ſideways over the other columns, till the heights are all equal to its own. The ſame is true of any other eminences or longer columns. Therefore, when all the ſubſidences are effected, and the maſs is at reſt, its form will be that of a ſolid, whoſe ſurface is every where equidiſtant from its center. And this is the property of a ſphere.

P This takes place in a maſs whoſe parts preſerve the ſame ſituation with reſpect to its center; but if the ſphere be cauſed to revolve on its own axis, a centrifugal force will be produced that will diminiſh the gravity of all its parts, except thoſe which are ſituated in the axis of rotation. This diminution will be greateſt in the equator, becauſe the velocity is there greateſt, and becauſe the centrifugal force acts directly againſt the force of gravity. And the nearer any parallel of latitude or circle of rotation is to one of the poles, the leſs will the gravity of the parts be affected; both the above mentioned cauſes being leſs. The equilibrium, before ſubſiſting between the columns in a ſpherical figure, will conſequently be deſtroyed, and the ſame effect muſt take place, as would have followed if the columns at the polar regions had been lengthened or augmented in maſs beyond thoſe near the equatorial parts, that is to ſay, the columns near the poles will ſpread over thoſe towards the equator, till the difference of their lengths compenſates for the difference of their gravities. Thus the ſphere, by its rotation, will be changed into a ſolid, whoſe radii, drawn to the center, are longeſt near the equator, and ſhorteſt towards the poles, the axis being the ſhorteſt of all its diameters.

By computation grounded on theſe and other conſiderations, it is ſhewn, that bodies at the equator of the Earth loſe more than $\frac{1}{230}$ part of their gravity, and that the equatorial diameter is to the axis as 231 to 230, upon the ſuppoſition that the Earth is every where of the ſame uniform denſity. For what has been ſaid of a fluid maſs will hold good of the Earth, ſince if it were not of this figure, but ſpherical, the ocean would overflow the regions near the equator, and leave the polar regions elevated many miles above the level of the ſea. But experience ſhews, that the land is in general no more elevated above the ſea in one part of the globe than another.

This decreaſe of gravity towards the equator is remarkably ſeen in the motion of pendulums.

For a pendulum, which in a higher latitude vibrates ſeconds, is found to go ſlower at the equator, and that in a much greater proportion than
can ariſe from the lengthening of the rod by heat,
nay, even in the coldeſt parts of the mountains of
Spaniſh America, which are conſtantly covered
v with ſnow. From the juſt mentioned quantity of
diminution of gravity, it is not difficult to compute the length of a pendulum (87, v) which ſhall
vibrate ſeconds in a given latitude, and from the
agreement of theſe computations with experience,
the oblate ſpheroidal figure of the Earth, as alſo
the diurnal rotation from which it originates, are
both confirmed.

v The ſame concluſion has likewiſe been obtained from the labours of many ingenious and learned men, who have actually meaſured the lengths of certain portions of the meridian in different latitudes, by which it appears, that the degrees are ſhorter towards the equator than nearer the poles. Whence it follows, that the meridian is more curved near the equator, and leſs near the poles, or in other words, that the Earth is flattened about the polar regions.

w The meaſure of a degree of the meridian, beginning at the equator, was found to be 56750 French toiſes, and the meaſure of a degree of the meridian cutting the arctic circle, was found to be 57422 French toiſes *.

---

* See De la Lande's Aſtronomie, § 2655, & ſeq. for a detail of the principal enterprizes on this intereſting ſubject.

These

X Theſe menſurations conſtitute the experimental proof of the Earth's rotation on its axis; for it is evident, that a centrifugal force cannot be produced but by an abſolute motion: and as the effects of this force are obſerved in the figure of the Earth, and not at all in the heavens, the motion of the Earth muſt be abſolute and real, and that of the heavens only relative and apparent.

Y The planet Jupiter revolves on its axis in leſs than ten hours; a rapidity which much exceeds that of the Earth; and its figure differs accordingly much more from that of a ſphere, its equatorial diameter exceeding its polar diameter, according to the obſervations of aſtronomers, as 13 to 14 (126).

Z It has alſo been already noticed, that a ſimilar phenomenon is ſeen in the planet Mars (126).

If a number of fluid bodies revolved about the A
Earth at equal diſtances from its center, they
would, by the action of the Sun, or any other
planet, be ſubject to irregularities of the ſame
kind as the Moon has been ſhewn to have in its
motion; that is, they would approach nearer, and
move ſwifter at the conjunction and oppoſition than
at the quadratures (221, I. 222, L). And if the B
number of bodies were ſo great as to become conti-
guous, and form a fluid ring or circle, the parts
of this ring would be affected in the ſame manner.
If it were inclined to the ecliptic, the nodes would
be ſtationary when in the conjunction or oppoſi-
tion (226, U) and be carried in a retrograde direc-

tion in the other revolutions (229, z), but moſt ſwiftly when they were ſituated in the quadratures (226, u). Its inclination would likewiſe vary in every revolution (230, b), and in a ſpace of time ſomewhat leſs than a periodical year would be diminiſhed and increaſed, by turns, twice (230, a. 231, f).

c Suppoſe this fluid ring to be of the ſame diameter as the Earth, to be placed in a cavity hollowed round the Earth at the equator, and to revolve in the ſame time and direction as the Earth does on its axis. Its motion would not then be uniform (237, b), but at the conjunction and oppoſition ſwifter than the ſurface of the Earth, and ſlower at the quadratures; conſequently, with reſpect to the ſurface of the Earth, it would ebb and flow like a ſea. For, by reaſon of the increaſed ſwiftneſs at the conjunction and oppoſition, and the retardation at the quadratures, the fluid, between the conjunction or oppoſition and the enſuing quadrature, would form a cumulus or heap, while a correſpondent defect would happen in the other quadrants preceding the conjunction and oppoſition.

d If this ring be now ſuppoſed to be frozen or converted into a ſolid, the flux and reflux will ceaſe, but the preceſſion of the nodes and the libratory increaſe and decreaſe of the inclination will remain (237, b). Suppoſe the ring to adhere to the ſurface of the Earth at the equator, inſtead of being admitted into a cavity; it will then communicate part of its motion to the Earth, the nodes of whoſe equator will recede, but with a much ſlower motion than thoſe of the ring would have receded, if it had not adhered to the Earth; and the obliquity or angle which the equator makes with the ecliptic, will be diminiſhed and increaſed alternately twice in a year.

The elevation of the equatorial parts of the Earth have the ſame effect as ſuch a ring would have; for the exceſs of matter in thoſe regions ſupplies its place.

Aſtronomers begin the year in the Spring, when the Sun is in that node of the equator, or equinoctial point at which the days begin to lengthen in the northern hemiſphere. Now it is plain (187), that if the equinoctial points had no motion, the Earth would complete one revolution in its orbit in the ſame time that the Sun employs in apparently paſſing from one of the equinoxes, and returning again to the ſame. But, becauſe of the retrograde motion, the line of the nodes of the equator, or diameter of the Earth which joins the equinoctial points, is brought to coincide again with the line which joins the centers of the Sun and Earth, before its periodical revolution is completed; and therefore the circle of the ſeaſons is performed in leſs time than the Earth's revolution in its orbit. The actions both of the Sun and Moon on the redundant matter in the equatorial regions tend to produce this motion, which is ſo ſlow, that a complete revolution will not be finiſhed in leſs than twenty-five thouſand years.

H This is called the preceſſion of the equinoxes, and is the reaſon that the fixed ſtars appear to advance in longitude about 50 ſeconds of meaſure in a year; whence it has happened, that ſince the time of Ptolemy, the zodiacal figures have advanced the greateſt part of a whole ſign: the conſtellation Aries being ſituate in that part of the ecliptic which is denominated from Taurus, Taurus in the place of Gemini, &c. I The difference between the natural year or period of the ſeaſons, and the periodical year, or time of the Earth's revolution in its orbit, is $26^m\ 34\frac{1}{2}^s$; for the natural year conſiſts of $365^d\ 5^h\ 48'\ 45\frac{1}{2}''$, and the periodical year of $365^d\ 6^h\ 15'\ 20'$.

K The ſideral year, or time employed by the Sun in returning to the ſame apparent poſition with reſpect to a fixed ſtar, is $365^d\ 6^h\ 9^m\ 11\frac{1}{4}''$. The difference between the periodical and ſideral year is occaſioned by the motion of the apſis of the Earth's orbit (216, K).

L The libratory variation of the inclination of the equator to the ecliptic is termed the nutation of the Earth's axis. The theory of attraction had aſcertained its exiſtence, long before aſtronomical obſervations were brought to a ſufficient degree of perfection to render it ſenſible. Its whole effect ſcarcely amounts to 18 ſeconds. It was firſt obſerved by Dr. Bradley[*].

---

[*] Phil. Tranſ. January, 1748.

CHAP.

CHAP. V.

OF THE TIDES.

M THOUGH the cauſe of the tides may be collected from what was ſaid in the laſt chapter; yet, as it is the only obvious inſtance we have of the mutual gravitation of the cœleſtial bodies, it will be proper to give a more particular explanation of it.

N If the Earth were every where covered with a deep ſea, it is plain, from the reaſons before recited (238, c), that the water would not, in the diurnal rotation, move with the ſame uniform velocity as the Earth. For, if the apparent diurnal revolution of the Moon be called a lunar day, and be divided into twenty-four equal parts or hours, the water ſituated near the meridian over which the Moon at any time is, will move ſwifter, and the water ſituated near the meridian ſix hours to the eaſtward or weſtward, will move ſlower: becauſe the water on each parallel of latitude may be conceived to be a fluid ring, and will be affected by the diſturbing force nearly in proportion to its diameter. The ſea, then, being accelerated at the meridian upon which the Moon is, and retarded at the meridian, that is 90° or 6 hours to the eaſtward, will be accumulated between the two places; its greateſt height being at the half diſtance, or meridian which the Moon has paſſed three hours. And on the other hand, the retardation at the quadrature to the weſtward, preventing the water from flowing as faſt as the acceleration, at the meridian at which the Moon is, carries it away, the ſea muſt of courſe be depreſſed between the two places, its greateſt depreſſion being at the half diſtance, or meridian at which the Moon will arrive in three hours. A ſimilar accumulation and diminution will happen at the places which are diametrically oppoſite to thoſe here deſcribed, o though not altogether ſo great (232, o). The diſturbing force of the Sun will act in a like manner, but leſs ſtrongly; for, though the Moon's attractive force be vaſtly leſs than that of the Sun, yet, becauſe its diſtance in compariſon to that of the Sun from the Earth is very ſmall, the forces with which it acts on different parts of the Earth will vary more conſiderably from paralleliſm and equality; and the irregularities in any ſyſtem, which ariſe from the actions of forces from without, are occaſioned (79, w), not by the whole actions of the forces, but only by their differences in quantity, or want of paralleliſm in direction.

p Thus, it is evident, that the ſea, as far as circumſtances allow, muſt in every place be raiſed to its greateſt and leaſt height, alternately twice in each lunar day. Being elevated once at three lunar hours after the Moon has paſſed the meridian of the place, and once at twelve hours after, or three hours after the Moon has paſſed the oppoſite part

of the ſame meridian; and at ſix hours after each
of theſe elevations its greateſt depreſſions follow.
This appears by the tides in the Atlantic ocean on Q
the weſtern coaſts of Europe and Africa, and in the
Pacific ocean on the open coaſts of Aſia and America,
where high-water always happens about the
third hour after the Moon has paſſed the meridian,
except where the motion of the ſea is ſomewhat
impeded by flats or ſhoals.

The effects of the diſturbing forces of the Sun R
and Moon are not ſeen diſtinctly, but compounding
with each other produce a motion which is different
from what would have ariſen from the ſingle
action of either luminary. At the time of the conjunction S
or oppoſition their effects are united, and the
tides are greateſt, being what are called Spring-tides.
When the Moon is in the quadrature, the T
Sun's action raiſes the water where the action of
the Moon depreſſes it, and depreſſes it where the
action of the Moon raiſes it: from the difference
of their actions therefore ariſes the leaſt, or, as they
are called, Neap-tides. And, becauſe the action U
of the Moon exceeds that of the Sun, high-water
follows neareſt the third lunar hour. At other times V
high-water ariſing from the lunar force would happen
on the third lunar hour, and that which ariſes
from the Sun's force on the third ſolar hour; but
the forces being compounded, produce a tide which
happens at ſome intermediate time, though always,
on account of the greater force, neareſt to the third
lunar hour. Conſequently, when the third ſolar W

hour precedes the third lunar hour, as is the caſe while the Moon is in the firſt and third quarters, high-water happens ſooner than the third lunar hour, and the contrary happens when the third lunar hour precedes the third ſolar hour, as in the
x ſecond and fourth quarters. It is to be noted, that no diſtinction is here made between the hour of the Sun or Moon's paſſing the meridian above the horizon, or beneath it; the effect being nearly the ſame with reſpect to the tides.

Y The effects of the diſturbing forces of the Sun and Moon depend likewiſe upon their reſpective diſtances from the Earth. For theſe effects are
Z greater at leſs diſtances. And therefore in winter, when the Earth is in its perihelium, the Sun being nearer, cauſes the ſpring-tides to be ſomewhat greater and the neap-tides ſomewhat leſs, than in
A the ſummer; and the Moon being each month in its perigeum, does then, in like circumſtances,
B cauſe greater tides than at other times; whence it happens, that if a great ſpring-tide happens when the Moon is in its perigeum, the next ſpring-tide will be leſs, becauſe the Moon will be then in its apogeum, or greateſt diſtance.

C The tides vary likewiſe in conſequence of the
D varying declinations of the Sun and Moon. For if either of theſe luminaries were ſuppoſed to be at the pole, it would neither accelerate nor retard the diurnal rotation of the water, but would occaſion a conſtant elevation at the poles, by diminiſhing the effect of gravity there, and a con-

ſtant depreſſion at the equator, from an oppoſite cauſe (219, c. 220, E). Therefore, no alternate riſe and fall of the water, or tide, would be produced. Conſequently, as the Sun and Moon decline towards the pole, they gradually loſe their effects, and the tides become leſs conſiderable. E When the Sun is in the equator, and the Moon at the tropic, or its greateſt declination, the tides are leſs than when the Moon is at the equator, and the Sun in the tropic: becauſe in the firſt caſe the Sun's influence is the greateſt poſſible, and the Moon's leaſt; and in the latter the Moon's influence is the greateſt poſſible, and the Sun's leaſt: and as the tides depend more upon the Moon's influence than that of the Sun, they are greateſt when its action is greateſt. F When the Sun and Moon are both in the equator, the ſpring-tides are the greateſt of any. G However, becauſe the Earth is nearer the Sun in winter than in ſummer, the greateſt autumnal ſpring-tides are generally later than the equinox; and the greateſt vernal ſpring-tides are generally before the equinox. H

I When the Moon declines either to the northward or ſouthward of the equator, one of the greateſt elevations of the water follows the Moon, and deſcribes nearly the ſame parallel of latitude as the Moon, by the diurnal motion, apparently deſcribes; and the other greateſt elevation being diametrically oppoſite, muſt, of courſe, deſcribe a parallel of latitude at an equal diſtance on the other ſide of the equator. The greateſt elevation,

which moves on the ſame ſide of the equator with
a given place, will come nearer to it than the op-
K poſite elevation; and therefore when the Moon
declines towards the ſame ſide of the equator, as
that on which the given place is ſituated; the day-
tides, or tides which happen while the Moon is
above the horizon, will be greateſt, and the night-
tides, or thoſe which happen while the Moon is
beneath the horizon, will be leaſt. And the con-
trary happens when the Moon declines to the other
L ſide of the equator. Thus, the elevation at high-
water is alternately greater and leſs; and the diffe-
rence is greateſt when the Sun and Moon both
deſcribe the ſame tropic, becauſe the oppoſite ele-
vations then deſcribe the tropics, which are the
fartheſt from each other of any two parallel circles
they can poſſibly deſcribe. This difference is
found to be about a foot at Plymouth, and fifteen
inches at Briſtol.

M If the actions of the Sun and Moon were to
ceaſe at once, the tides would not immediately ceaſe,
but would continue for ſome time by the undu-
lating motion of the water. This undulation would
be greater, if the actions were to ceaſe at the time
N of a great tide than at the time of a leſs; and there-
fore leſs tides, which ſucceed greater, are more in-
creaſed by it than greater tides which ſucceed leſs:
conſequently the difference between the tides is ren-
dered leſs than it would otherwiſe have been, and
the greateſt ſpring and neap-tides do not happen
preciſely at the conjunction or oppoſition, and qua-
dratures, but two or three tides later.

If the greateſt acceleration and retardation of the diurnal motion (241, N) cannot ſubſiſt in the ſame ſea at the ſame time, the accumulation or defect muſt conſequently be leſs; that is to ſay, if one of the ſhores or coaſts of any ſea be leſs than ninety degrees to the eaſtward or weſtward of the other, and the eaſtern coaſt, for inſtance, be immediately under the Moon, the acceleration will, by cauſing the water to riſe, occaſion a defect or fall to the weſtward, becauſe the weſtern parts, being retarded, do not follow with a velocity ſufficient to ſupply what is carried to the eaſtward by the acceleration: and the greater this retardation the greater the defect or fall. But ſince by the ſuppoſition the weſtern ſhore is not 90 degrees diſtant, the retardation is not there ſo great as it would have been had the ſea been wider; and therefore the fall in that ſea is not ſo great. By a like argument it appears, that when the Moon is at the meridian of the weſtern coaſt the elevation is leſs, if the ſea be leſs than ninety degrees from eaſt to weſt. Hence in ſmall inland ſeas the tides are P inconſiderable; and for this cauſe, in the Atlantic ocean the tides do not riſe ſo high between the tropics as they do farther to the north or ſouth, the ſea being narrower between America and Africa in the lower than in the higher latitudes. From hence Q alſo follows the reaſon why the tides are ſo ſmall as they are found to be at the iſlands of St. Helena and Aſcenſion, which lie in the middle of that ſea; for, ſince the water cannot riſe on the one ſhore but by falling at the other, it muſt continue at a mean height at theſe intermediate diſtant iſlands.

R This theory of the tides is perfectly conſentaneous to experience in the open and deep oceans; and in the leſſer ſeas, as has been obſerved, the tides are very ſmall. But, when thoſe leſs ſeas have a free communication with the ocean, the tide flows into them in a kind of wave, which on its arrival at
S any place cauſes high-water. Thus it is high-water in the ocean to the weſtward of England and Ireland at the third lunar hour; after which it begins to ſubſide. This ſubſidence muſt, of courſe, raiſe the water round about, whence a flood begins to enter the Engliſh channel at about the ſixth hour; its courſe being retarded by the ſhallowneſs of the water. Another flood enters the German ſea to the northward, near the Orkney iſlands, and proceeds to the ſouthward. As theſe floods proceed on their reſpective courſes, it is high-water ſucceſſively at every place on the coaſts at which they arrive, and when the wave has paſſed any place the water be-
T gins there to ſubſide. For example; it is high-water at Plymouth about the ſixth hour, at the Iſle of Wight about the ninth hour, and at London-bridge about the fifteenth hour after the Moon has paſſed the meridian, and cauſed the tide in queſtion. Therefore, when it is high-water at Plymouth the water out at ſea has half ſubſided; when it is high-water at the Iſle of Wight it is low-water out at ſea; and when it is high-water at London-bridge it is low-water at the Iſle of Wight, and a ſecond flood or elevation has already come to its height out at ſea.

There are ſituations where the tide may be car- u
ried to the ſame port by different paſſages or chan-
nels, and may paſs quicker through one paſſage
than another: in which caſe, the ſame tide, arriv-
ing at different times through the different paſſages,
muſt occaſion a variety of phenomena. Suppoſe v
two equal tides to arrive at the ſame port from dif-
ferent places; the one at the third, and the other at
the ninth hour after the Moon has paſſed the meri-
dian; the firſt tide therefore preceding the latter by
ſix hours; and ſuppoſe the Moon to be at the equa-
tor: then, every ſix hours a tide will arrive, which,
flowing in at the ſame time as the preceding equal
tide ebbs out, will cauſe the water to continue at
the ſame height; and thus it will neither riſe nor fall
during the whole day. But, if the Moon decline
from the equator, the tides in the ocean will be
alternately greater and leſs, as has been obſerved;
and therefore there will arrive at this port, alter-
nately, two greater floods proceeding from the
greater tide in the ocean, and two leſs floods pro-
ceeding from the leſſer tide in the ocean. At the
mean or intermediate time between the arrivals of
the two greater tides, the water will then be higheſt;
between a greater and a leſs tide it will be at a
mean height; and loweſt of all at the middle time
between the arrivals of the two leſs tides. By theſe
means, in the ſpace of twenty-four hours, the ſea
will riſe to its greateſt, and fall to its leaſt height
but once, inſtead of twice, as in general it does in
other places; and if the Moon decline towards the
ſame

ſame ſide of the equator, as that on which the port is ſituated, the two greater tides will arrive at the third and ninth hours, and the greateſt elevation will be at the ſixth hour, or at about the ſetting of the Moon: the leaſt elevation will conſequently happen between the two leaſt tides, at the eighteenth hour, or about the riſing of the Moon. And the ſame effects will take place when the Moon declines to the contrary ſide of the equator; but with this difference, that whereas high and low water happened then reſpectively at the ſetting and riſing of the Moon, they will in the preſent caſe happen reſpectively at the riſing and ſetting of the Moon.

w A remarkable inſtance of all theſe particulars is adduced by Dr. Halley, in the port of Batſha, in the kingdom of Tonquin, which lies in 20° 50′ north latitude. There, on the day on which the Moon paſſes the equator, the water ſtagnates; afterwards, on the Moon's declining to the northward, it begins to ebb and flow; not twice in the day, as in other ports, but once only; and high-water happens at the ſetting, and low-water at the riſing of the Moon. The tides increaſe with the Moon's declination for ſeven or eight days; after which, for the next ſeven days, they decreaſe by the ſame gradation as they before increaſed, till the Moon's arrival again at the equator, when they ceaſe, and upon its changing its declination are reverſed. For while its declination becomes ſoutherly, low-water happens at the ſetting, and high-water at the riſing of the Moon; which continues till its declination again changes. Now it appears, that the tide muſt come to this port by two inlets or paſſages; one between the continent of China and the iſland Luconia, communicating with the Chineſe ocean, and the other between the iſland of Borneo and the continent. But whether the tide arriving from the Indian ocean, after a courſe of twelve hours, and from the Chineſe ocean after a courſe of ſix, and thence happening on the third and ninth lunar hours, be the cauſe of theſe appearances; or whether ſome other circumſtances may not be concerned in producing the effect, muſt be determined by obſervations on the neighbouring coaſts.

BOOK II.

SECT. I.

Of Light.

CHAP. I.

CONCERNING THE MOTION OF LIGHT.

IN the former part of this Treatiſe our attention has been employed in conſidering ſuch effects as ariſe from the motions and mutual actions of bodies, of magnitude ſufficient to become the objects of our ſenſes. It may be eaſily ſeen, that phenomena of this kind are but few compared with the very great number of operations which ariſe from the actions of bodies too minute to be determined but by deductions or inferences made from their effects. We have contemplated the great outline of the univerſe, and its vaſtneſs cannot but excite the aſtoniſhment of creatures who are deſtined at preſent to occupy an exceedingly ſmall part of it. As we proceed to examine that ſmall part, we ſhall develope a ſcene of another kind, which, though expanded through a leſs portion of ſpace, is equally immenſe and unlimited with regard to the field it affords for admiration and perpetual diſcovery.

The ſucceſs of every enquiry depends in no ſmall degree on the order employed in the ſeveral reſearches to be made. It is obvious, that the probability of error is greater the more complicated the ſubject; whence the neceſſity of firſt examining the moſt ſimple, and thence proceeding to more compounded objects, is evident. This principle leads us, in our conſideration of the particular properties of various bodies, to attend firſt to thoſe of light.

Z It is generally agreed, that light conſiſts of ſmall bodies or particles, projected with great velocity in all directions, from the luminous or radiant body. No ſolid objections have been made to this hypotheſis, which appears to be more ſimple than any other, and is perfectly conſiſtent with all the phenomena yet obſerved; and theſe are ſo many and ſo various as to leave very little doubt of its truth.

A The velocity of light was firſt determined by Monſ. Romer, from obſervations on Jupiter's Moons (140, w) and the meaſure deduced from his obſervations was afterwards confirmed and eſtabliſhed by the diſcovery of the aberration of the fixed ſtars. The principles on which this diſcovery is founded may be familiarly explained as follows *.

B Suppoſe a tube to be erected perpendicular to the horizon, at a time when it rains, the drops falling perpendicularly down, and ſuppoſe the diameter of the tube to be ſuch as to admit but one drop at a time: then it is plain, that if a

---

* The original account of this diſcovery may be ſeen in a paper by its inventor Dr. Bradley, inſerted in the Philoſophical Tranſactions for the year 1728, No. 406.

drop of water enter the orifice of the tube it will fall to the bottom without touching its ſides. But if the tube, without altering its perpendicularity, be moved along in the direction of the horizon, any drop that enters will ſtrike againſt one of its ſides, and none will paſs clear through while the motion continues, unleſs the upper end of the tube be alſo inclined towards the part to which its motion is directed.

Thus, if A B (fig. 70) repreſent the horizon, C D the perpendicular tube, and G D the courſe of a drop of rain: then, if C D be moved towards A, while the drop is falling within the tube, it is evident that the inner ſurface of the tube, which is ſituated towards B, will be carried againſt the drop, and prevent its arriving at the bottom without touching. But if the inclined tube E C be moved with a ſimilar motion to that of the drop from E to D, in the time that the drop moves from C to D, the lower orifice of the tube and the drop will be found at D at the ſame inſtant; and the velocity of the drop will be expreſſed by C D, and that of the tube by E D.

The ſame reaſoning holds good, if inſtead of drops of rain we ſuppoſe particles of light, and a teleſcope inſtead of a tube. For to an obſerver, who through the tube C D views the vaſtly diſtant object G, if the motion of light be inſtantaneous, or infinitely ſwift, no finite motion of C D, its poſition being unaltered, can prevent its being viſible; ſince, by the ſuppoſition, the light which enters at

C will arrive at D before C D can have moved at all. But if light be propagated in time, and the obſerver be carried by a motion ſimilar, as to acceleration, to that of light, the tube muſt be inclined to the ray in an angle, whoſe ſine is to the ſine of C E D, or the angle the tube makes with the line of the obſerver's motion, as the velocity of the obſerver is to the velocity of light. For in the triangle DCE, the ſides D E and D C, which expreſs theſe velocities, are as the ſines of their oppoſite angles. Hence if the angle of the inclination of the tube to the ray of light, together with the velocity and direction of the obſerver's motion be known, the velocity of light may be determined.

E By this theory, which is eſtabliſhed by a great number of obſervations on ſtars of different magnitudes and ſituations, it appears, that the ſmall apparent motion the fixed ſtars have about their real places, which is called their Aberration, ariſes from the proportion which the velocity of the Earth's motion in her orbit bears to that of
F light. This proportion is found to be as 10210 to 1: from whence it follows, that light moves or is propagated as far as from the Sun to the Earth
G in 8′ 12″. And it likewiſe appears, that the velocity of light is uniform, and the ſame, whether original, as from the ſtars, or reflected, as from the ſatellites of Jupiter (140, W.)

H The velocity of light being known, an eſtimate might be made of the magnitude of its particles, if we were in poſſeſſion of good obſervations of the effects of their momentum. For example, it is found, that a ball from a cannon at its firſt diſcharge flies with a velocity of about a mile in * eight ſeconds, and would therefore arrive at the Sun in thirty-two years, ſuppoſing it to move with unremitted velocity. And light, as was before obſerved, moves through that ſpace in about eight minutes, which is two million times as faſt. But the force with which bodies move are as their maſſes multiplied by their velocities (19, L); and conſequently if the particles of light were equal in maſs to the two millionth part of a grain of ſand, we ſhould be no more able to endure their impulſe than that of ſand ſhot point blank from the mouth of a cannon. I K

From ſeveral experiments † that well deſerve to be repeated, in which the Sun's rays were thrown, much condenſed, upon a very light lever, ſuſpended ſo as to be removeable horizontally, it appears that the momentum of light is great enough to be rendered ſenſible by the velocity it communicates to bodies by its impulſe. L

The rarity of light is not leſs a matter of wonder at the firſt conſideration, than its velocity and the minuteneſs of its particles. For its rays croſs each other in all directions without the leaſt apparent diſturbance. We can eaſily ſee through a ſmall M

---

* This varies according to the charge of powder and other circumſtances.

† By Mr. Michell. See Prieſtley's Optics, p. 387.

hole,

hole, not exceeding the $\frac{1}{100}$ part of an inch, the objects, as the ſky, trees, houſes, &c. which occupy almoſt an entire hemiſphere. The light proceeding from all theſe objects muſt therefore paſs at the ſame time through the hole in a very great variety of directions before they arrive at the eye; yet it does not appear that viſion is in the leaſt diſturbed by that means.

N This is, however, explained without difficulty from a well-known circumſtance relative to the organs of viſion. For the action of light produces an effect on the eye that is not inſtantaneous but laſts a conſiderable time. Suppoſe the effect of light on the eye to continue without ſenſible diminution, after the light has ceaſed to act, for the $\frac{1}{100}$ part of a ſecond, and it will follow that a ſucceſſion of particles of light arriving at the eye, by equal intervals, to the number of three hundred in a ſecond, will be ſufficient to excite a conſtant and uniform ſenſation of the preſence of light. And ſince the velocity of light is ſuch that it paſſes through about one hundred and ſeventy thouſand geographical miles in a ſecond (140, W); this ſpace divided by 300, will give nearly 570 miles for the diſtance between each of the above-mentioned ſucceſſive particles. It is not therefore to be wondered, that the particles of light do not interrupt each other, when we attend to their extreme minuteneſs (257, K), and the very great diſtance at which they may follow each other without preventing the conſtancy of their effect.

That the effect of light on the eye remains for a time is evinced from ſeveral obſervations. We are continually ſhutting our eyes, or winking, and ſhould on thoſe occaſions loſe ſight of all the ſurrounding objects, if the effect of their light did not continue during the time the eye is ſhut. Again, if a ſtick, or any other object, be whirled round in a circle, there is a certain velocity beyond which the object will fill the whole circle. This experiment is vulgarly made with a lighted firebrand, and obviouſly ſhews that the impreſſion, made on the eye by the light of the firebrand, when in any given point of the circle, is ſufficiently laſting to remain till the firebrand has deſcribed the whole circle, and again renews its effect, which is by that means rendered continual and uniform. Every one muſt have been ſenſible of the ſtrong and laſting impreſſion the Sun's direct light makes upon the eye; and impreſſions of the ſame kind from other objects, though weaker, are much leſs ſo than is generally imagined. o

With reſpect to the duration of the impreſſions of light, it has been obſerved, that the teeth of a cog-wheel in a clock* were ſtill viſible in ſucceſſion when the velocity of rotation brought 246 teeth through a given fixed point in a ſecond. In this caſe it is clear, that if the impreſſion made on the eye by the light reflected from any tooth, had laſted without ſenſible diminution for the 246th part of a ſecond, the teeth would have formed one p unbroken line, becauſe a new tooth would have continually arrived in the place of the anterior one before its image could have diſappeared. If a live coal be whirled round, it is obſerved†, that the luminous circle is complete when the rotation is performed in $\frac{8}{60}$ of a ſecond. In this inſtance we ſee that the impreſſion was much more durable than the former. Laſtly, if an obſerver ſitting in a room direct his ſight through a window, to any particular object out of doors, for about half a minute, and then ſhut his eyes, and cover them with his hands, he will ſtill continue to ſee the window, together with the outline of the terreſtrial objects bordering on the ſky. This appearance will remain for near a minute, though occaſionally vaniſhing and changing color, in a manner that brevity forbids our minutely deſcribing. From theſe facts we are authorized to conclude, that all impreſſions of light on the eye laſt a conſiderable time; that the brighteſt objects make the moſt laſting impreſſions; and that, if the object be very bright, or the eye weak, the impreſſion may remain for a time ſo ſtrong, as to mix with and confuſe the ſubſequent impreſſions made by other objects. In the laſt caſe the eye is ſaid to be dazzled by the light.

---

* Watſon on Time.

The ſpace through which light paſſes is called a medium; by which term reference is had to the quantity or denſity of the matter contained in the ſpace: thus, glaſs and air are mediums, but a vacuum, or ſpace abſolutely void, is a medium alſo.

---

† By M. D'Arcy. See Mem. de l'Acad. des Sciences, 1765; or Prieſtley's Optics, p. 634.

When light paſſes through mediums, either abſolutely void, or containing matter of an uniform denſity, and of the ſame kind, it always is found to proceed in ſtraight lines. Whence it follows, that the rarity of light increaſes as the ſquare of the diſtance from the radiant body. For the light which falls on the ſquare A B C D, (fig. 71.) from the point R, at the diſtance R A, will be ſpread over a ſurface, a b c d, four times as large at twice the diſtance, or R a. R

C H A P. II.

OPTICAL DEFINITIONS AND PRINCIPLES.

WHEN a ray of light paſſes out of one medium into another, and is bent out of its courſe at their common ſurface, this bending is called refraction. S

When a ray of light proceeds to the common ſurface of two mediums, and inſtead of paſſing from the one into the other, is turned back into the firſt, this turning back is called reflection. T

The angle of incidence is the acute angle which the line deſcribed by the ray of light makes with a line drawn perpendicular to the ſurface at the point of incidence. U

The angle of reflection or refraction is the acute angle, which the line deſcribed by the ray of light V after reflection or refraction makes with the perpendicular to the ſurface at the point of incidence.

W Thus, if R S repreſent the common ſurface of two mediums, A C (fig. 72.) a ray of light incident at C, and P Q a line interſecting the ſurface at right angles at C; then the angle A C P is the angle of incidence. If it be reflected at C, ſo as to return in the line C B, then the angle P C B is the angle of reflection: and if it be refracted at C, ſo as to proceed in the line C F, the angle Q C F is the angle of refraction.

X The angles of incidence, reflection, and refraction lie in one and the ſame plane.

Y The angle of reflection is equal to the angle of incidence.

Z If the refracted ray be returned directly back to the point of incidence, it ſhall be refracted into the line which was before deſcribed by the incident ray.

A The refractive powers of different mediums are nearly as their denſities; that is to ſay, if a ray of light paſs out of a rarer into a denſer medium, it will in general be refracted towards the perpendicular, ſo that the angle of refraction will be leſs than the angle of incidence. And the contrary will happen if the ray paſs out of a denſer into a rarer medium (262, Z).

B The ſine of the angle of incidence is either accurately or very nearly in a given ratio to the ſine of the angle of refraction, in all obliquities of the incident ray.

All

C All objects ſeen by reflection or refraction appear in that place or direction, from whence or in which the rays were laſt reflected or refracted to the eye.

D Thus, if the ray A C (fig. 72.) proceed from an object at A to C, and be thence reflected to the eye of a ſpectator at B, the object will be ſeen not at A but at T, in the direction of the reflected ray B C. And if the ray F C proceed from an object at F, and be refracted into the direction C A to the eye of a ſpectator at A, the object will be ſeen not at F but at N, in the direction of the refracted ray A C.

E On this account it is that objects are ſeen in mirrors or looking-glaſſes, and that objects ſeen under water appear out of their true places. Let A B C D (fig. 73.) repreſent a veſſel containing water, whoſe ſurface is F G, and let O repreſent an object at the bottom. Then, to an eye at E the object O will be ſeen at K, by means of the ray O L, which paſſing from a denſer to a rarer medium (262, A), is refracted from the perpendicular P Q into the direction L E. Or let A B C D (fig. 74.) repreſent a veſſel ſo placed with reſpect to the candle E, that the ſhadow of the ſide A C may fall at D. Suppoſe it now filled with water, and the ſhadow will withdraw to d, the ray of light E A, inſtead of proceeding to D, being refracted to d. And there is no doubt but that an eye placed at d would ſee the candle at e in the direction of the refracted ray d A.

F The foregoing principles are founded on experiment or obſervation, and the mathematical application of them to the rays of light which paſs through glaſſes, or are reflected from mirrors of various figures, conſtitutes that branch of the ſcience of optics which teaches the conſtruction of inſtruments. But it will be proper to avoid the explanation of theſe for the preſent, till an account has been given of the various reflexibility, refrangibility, and colours of light.

CHAP. III.

OF THE VARIOUS REFRANGIBILITY OF THE RAYS OF LIGHT.

G LIGHTS which differ in colour differ alſo in refrangibility, and the contrary.

H Let AB (fig. 75.) repreſent a wedge or triangular priſm of glaſs, then the triangle ABC (fig. 76.) may be conceived to be a ſection of the ſame, at right angles to its axis. Suppoſe JN to be a ray of light incident at N, and thence refracted to E, on the ſurface CB, where it is again refracted into the direction EM; ſuppoſe in to be another ray parallel to the former, and conſequently incident at n, with the ſame angle. Now, if the ray in have exactly the ſame capability or diſpoſition to be refracted by the priſm, as the ray JN, the angles of refraction will alſo be equal, and in will, when refracted into the directions n e and e m, ſtill continue parallel to the ray J N, which is refracted into N E and E M. But if it be more refrangible it will be refracted into directions, as n f and f g, verging more towards the baſe A C, or, if leſs refrangible, it will be refracted into directions, as n h and h k, that verge leſs towards the baſe A C. Whence it appears, that if a pencil or collection of rays fall parallel to each other on one of the ſides of a priſm, and do not proceed parallel to each other on their emergence, it muſt be becauſe ſome of the rays are more refrangible than others.

Let the ſpace contained between E G and M R (fig. 77.) repreſent a darkened chamber, of which thoſe lines repreſent the ſides. Let J N repreſent a pencil of light from the Sun, paſſing through the hole F, and incident on the ſide B C of the priſm A B C. It is plain, that if the priſm were not interpoſed, the pencil J N would proceed to S, and there illuminate a ſmall circular ſpot on the wall; and from the preceding explanation it is evident, that if all the rays of light be equally refracted by the priſm, the whole pencil, being equally turned out of its courſe, will ſuffer no alteration with reſpect to the paralleliſm of its rays, and conſequently will, after refraction, proceed to Q, and there illuminate a ſpot ſimilar to that which would have appeared at S. But if the pencil be compoſed of rays not alike refrangible, the moſt refrangible rays will be thrown farther from S, and

the leaſt refrangible, being leſs deflected out of
their courſe, muſt fall on a part of the wall nearer
to S, while thoſe which are refrangible in the intermediate
degrees will fall at interpoſed diſtances
K between them. The actual experiment determines,
that the Sun's light is compoſed of rays,
whoſe refrangibilities are not all the ſame; for
after emerging from the priſm, inſtead of illuminating
a circular ſpace, they are ſpread into a long
ſpectrum, bounded by right-lined ſides and circular
ends, and whoſe length is at right angles to
the direction of the axis of the priſm.

L This oblong ſpectrum is variouſly coloured. The lower part, which conſiſts of the leaſt refrangible rays, is of a lively red, which, higher up by inſenſible gradations, becomes an orange; the orange in like manner is ſucceeded by a yellow; the yellow by a green; the green by a blue; after which follows a deep blue or indigo; and laſtly, a faint violet. With a priſm, whoſe refracting angle was 63½ degrees, ſo placed, that the angle of incidence on the firſt ſurface was equal to the angle of refraction at the emergence or ſecond ſurface, the ſpectrum, received on a wall at the diſtance of 18½ feet, was 10 inches, or 10⅛ in length. Its breadth is in all caſes equal to that of the circle, which would have been formed at that diſtance by the admitted beam or pencil of light, if the priſm had not been interpoſed.

M The ſpaces occupied by the ſeveral colours of the ſpectrum anſwer to the ſubdiviſions of a muſical

UNIV. OF MICH.

ſical chord: thus, if AGMF (fig. 79.) repreſent the ſpectrum, and the lines FM, ba, dg, zy, &c. mark the confines of the colours; the ſpace Mabf being occupied by the red, agdb by the orange, gyzd by the yellow, and ſo forth, in the order above-mentioned, and GM be prolonged to x, ſo that MX may be equal to GM, the lines GX, lx, ix, ex, yx, gx, ax, MX, will be in proportion to each other as the numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, and therefore expreſs the chords of the key, tone, leſs third, fourth, fifth, greater ſixth, greater ſeventh, and octave. Or, to render it more familiar to thoſe whoſe knowledge of muſic is merely practical, let AB repreſent the ſtring of a violin or guittar, and a fret or ſmall bridge be fixed on the finger-board at the middle diſtance a, between A and B; and likewiſe frets at g, f, e, d, c, b, ſo that the diſtances ag, gf, fe, ed, dc, cb, and bA, may be in proportion to the ſpaces occupied reſpectively by the red, orange, yellow, green, blue, indigo, and violet in the ſpectrum; then, if the open ſtring ſound A, the regular aſcent of the ſtopt notes will be in the minor third from that key; the notes being A, B, C, D, E, F ſharp, G ſharp, and A octave.

Thoſe rays which have the ſame degree of refrangibility will, after refraction, fall within a circle equal to that which would have been illuminated by the light if ſuffered to proceed to S (264, H): and therefore the ſpectrum may be conceived to be compoſed of an indefinitely great N

number of ſuch equal circles, whoſe centers are all on the ſame line. For example; if ABDC (fig. 78.) repreſent the ſpectrum, the circle AB being formed by the red, or leaſt refrangible rays, and the circle DC by the violet, or moſt refrangible rays, then the rays, whoſe refrangibility is intermediate, will form an innumerable ſeries of circles, and fill up the whole ſpace, ſo that AC and BD will appear as right
O lines. Now, it is obſervable, that though the light in the ſpectrum thus ſeparated into its original rays is much leſs compounded than before, yet it is ſtill compounded in no ſmall degree by the interference of the circles with each other, particularly at the line EF, equidiſtant between AC and BD; and that at the lines AC and BD, where the circles do not interfere at all, the light is perfectly homogeneous
P or uncompounded. But becauſe the colours in the ſpectrum contiguous on either ſide of any given colour do by mixture compound a colour that differs inſenſibly from the original intermediate colour itſelf, a right line drawn perpendicularly acroſs the ſpectrum will be found in the ſame colour
Q throughout. For moſt experiments in which uncompounded light is required, that of the ſpectrum will be found ſufficiently ſo, but in caſes where a greater nicety is demanded, the common diameter of the circles, or breadth of the ſpectrum, may be diminiſhed by making the hole at F ſmaller, or the figure of the hole may be altered.

R It is evident from what has already been ſaid, that this phenomenon ariſes from the nature of light itſelf,

ſome of the rays of which are more refrangible than
others. And as an additional confirmation it is s
obſerved, that if the ſpectrum be received on a
board which is perforated, ſo as to let paſs one ray
of light, or colour, that ray will not be changed
by any refraction it may be afterwards made to
ſuffer, but continues the ſame both in colour and
refrangibility. And if the colours of the ſpectrum T
be any reflection or refraction made to unite again,
they will again form the compounded colour of
whiteneſs.

The quantity of the diſperſion of the rays of u
light, which at equal diſtances from the priſm is
nearly expreſſed by the length of the ſpectrum,
does not follow the quantity of the refraction of the
mean ray, except in mediums of the ſame kind.
Thus, if two priſms of different kinds of glaſs re- v
fract the ſolar ray equally out of its firſt direction,
the ſpectrum of colours formed by the one will be
much longer than that formed by the other; and
it is found, that in equal angles of mean refraction, w
glaſs, in the compoſition of which much lead enters,
diſperſes the light into its component colours much
more than glaſs which abounds with alkaline ſalts *.

If by means of two priſms, a ſmall piece of pa- x
per be illuminated, the one half with red, and the
other half with violet light, and an obſerver view
the ſame through another priſm, the paper will, by
the different refrangibility of the rays, appear di-

---

* Acta (or perhaps Miſcellanea) Berolinenſia, for 1766; quoted by Prieſtley in his Optics, p. 474.

vided into two. For the violet half being ſeen by a more refrangible light, will appear to be carried farther from its true place than the red, and will Y therefore ſeem to be ſeparated from it. The ſame is likewiſe true of colours which ariſe from the ſeparation of light which is made by bodies on which it falls, and which we are apt to call natural colours; for if a paper be painted, the one half with a lively red, and the other half with an indigo, and it be placed in the Sun's light, it will in the ſame manner appear divided, if viewed through a priſm.

CHAP. IV.

OF THE VARIOUS REFLEXIBILITY OF THE RAYS OF LIGHT.

Z THE Sun's light conſiſts of rays which differ in reflexibility, and thoſe rays which are more refrangible are alſo more reflexible than others.

A Let A B C (fig. 80.) repreſent a priſm, whoſe angle B is a right angle, and the two angles A and C equal to each other. Suppoſe J N to be a beam of light which paſſes through the ſurface B C, and is incident on A C at N. It will then emerge in the direction N G, ſo that the ſine of the angle of refraction G N W may be in a certain ratio (262, B) to the ſine of the angle of incidence B N Z, which in glaſs is as 3 to 2, nearly. Now, when the angle of incidence at N is ſuch, that the ſine of the angle of refraction is equal to the radius, the angle of refraction becoming a right angle, the ray cannot emerge, but will be totally reflected or turned back into the glaſs. This happens in glaſs when the angle of incidence is about 41 degrees.

That the component rays of light are not at all equally diſpoſed to be reflected, is proved by turning the priſm ſlowly on its axis, till the light begins to be reflected; for it then appears, that the more refrangible rays are reflected ſooner, or at leſs angles of incidence than thoſe which are leſs refrangible. Let N M repreſent the reflected beam, and ſuppoſe the priſm V X Y placed ſo as to receive and ſeparate it into its component colours by refraction: then the light which firſt begins to be reflected, conſiſting almoſt intirely of violet rays, will by the ſecond priſm be refracted ſo as to fall at p, and paint a violet colour. As the firſt priſm continues to be turned on its axis, the light is more and more copiouſly reflected, and the colours between p and t appear in ſucceſſion according to their order in refrangibility; violet, indigo, blue, green, yellow, orange, and laſtly red, at which time the reflection becomes total: the colours formed by refraction at M O diſappearing as thoſe at p t appear. B C

White light being proved to conſiſt of rays which differ in refrangibility, reflexibility, and the power of exciting the idea of colour; it is clear that nothing more is neceſſary to account for the colours of bodies than to ſuppoſe each body endued with a power or aptitude to reflect the rays of one particular colour, and to imbibe the reſt. D

But the truth of this does not reſt on mere ſuppoſition.
E Bodies expoſed to the uncompounded light
of the ſpectrum, are ever found to be of the colour
of the light in which they are placed, with this only
difference, that they appear much more lively in
that colour, which is the ſame with that which they
F exhibit in the day light. And from hence it appears,
that the colours of bodies cannot be ſo
homogeneous and full as thoſe of the ſpectrum;
for ſince they reflect all colours in ſome degree as
well as the principal or predominant one, that
principal colour muſt be much diluted and weakened
G by the mixture. It may likewiſe from hence
be inferred, that as the uncompounded colours are
not changeable by refraction, ſo neither are they
changeable by reflection.

H Language being invented chiefly for the expreſſion
of ordinary events that do not require any
great precision, it very frequently happens, that
the ſame word is uſed to denote very different
things. It is proper to be remarked, that the word
colour is thus uſed. If the word be uſed to denote
the ſenſation or idea excited in the mind, it is ſufficiently
obvious, that it cannot be ſcientifically
uſed to denote that attribute by which bodies are
able, by reflecting the rays of light, to produce the
ſenſation. And ſtill leſs ought it to be uſed to
imply that quality the various kinds of light poſſeſs,
of producing the ſenſation, when ſeparated from
each other, either by reflection from bodies or
otherwiſe. It may, however, be allowed to uſe

the terms coloured rays, or coloured bodies, though the ſenſation of colour, the ſpecific properties of the rays, and of the reflecting bodies, are undoubtedly things very different from each other. So different, indeed, that this remark might with juſtice be ſuppoſed unneceſſary, if experience had not ſhewn, that among the pretenders to philoſophical knowledge ſome have been found capable of miſtaking in this very particular *.

CHAP. V.

CONCERNING THE RAINBOW.

THE inſtance of the ſeparation of the primary colours of light which ſeems moſt remarkable, is that of the rainbow. It is formed in general by the reflection of the rays of the Sun's light from the drops of falling rain, though frequently it appears among the waves of the ſea, whoſe heads or tops are blown by the wind into ſmall drops, and is ſometimes ſeen on the ground when the Sun ſhines on a very thick dew. Caſcades and fountains, whoſe waters are in their fall divided into drops, exhibit rainbows to a ſpectator, if properly ſituated during the time of the Sun's ſhining; and water blown violently out of the mouth of an obſerver, whoſe back is turned towards the Sun, never fails to produce the ſame phenomenon. This appear- I K L

---

* The oppoſers of Newton's diſcoveries on light and colours have falſely affirmed, that he taught that the rays of light were coloured.

appearance is alſo ſeen by moonlight, though ſeldom vivid enough to render the colours diſtinguiſhable; and the artificial rainbow may be produced even by candle-light on the water which is ejected by a ſmall fountain or jet d'eau. All theſe are of the ſame nature, and dependant on the ſame cauſes, an idea of which may be formed by the following conſiderations.

M Let the circle W Q G B (fig. 81.) repreſent a globe or drop of water upon which a beam of parallel light falls, of which let T B repreſent a ray falling perpendicularly at B, and which by conſequence (262, Y, B) either paſſes through without refraction, or is reflected directly back from Q. Suppoſe another ray I K, incident at K, at a diſtance from B, and it will be refracted according to a certain ratio (262, B) of the ſines of incidence and refraction to each other, which in rain water is as 529 to 396, to a point L, whence it will be in part tranſmitted in the direction L Z, and in part reflected to M, where it will again in part be reflected, and in part tranſmitted in the direction M P; being inclined to the line deſcribed by the incident ray in the angle I O P. Another ray A N, ſtill farther from B, and conſequently incident under a greater angle, will be refracted to a point F, yet farther from Q, whence it will be in part reflected to G, from which place it will in part emerge, forming an angle A X R with the incident A N, greater than that which was formed between the ray M P and its incident ray. And thus, while the angle of incidence or diſtance of the point of incidence from B increaſes, the diſtance between the point of reflection and Q, and the angle formed between the incident and emergent reflected rays will alſo increaſe; that is to ſay, as far as it depends on the diſtance from B: but as the refraction of the ray tends to carry the point of reflection towards Q, and to diminiſh the angle formed between the incident and emergent reflected ray, and that the more the greater the diſtance of the point of incidence from B, there will be a certain point of incidence between B and W, with which the greateſt poſſible diſtance between the point of reflection and Q, and the greateſt poſſible angle between the incident and emergent reflected ray will correſpond. So that a ray incident nearer to B ſhall, at its emergence after reflection, form a leſs angle with the incident, by reaſon of its more direct reflection from a point nearer to Q; and a ray incident nearer to W, ſhall at its emergence form a leſs angle with the incident, by reaſon of the greater quantity of the angles of refraction at its incidence and emergence. The rays which fall for a conſiderable ſpace in the vicinity of that point of incidence with which the greateſt angle of emergence correſponds, will, after emerging, form an angle with the incident rays differing inſenſibly from that greateſt angle, and conſequently will proceed nearly parallel to each other; and thoſe rays which fall at a diſtance from that point will emerge at various angles, and conſequently will diverge. Now, to a ſpectator, whoſe back is turned towards the radiant body, and whoſe eye is at a conſiderable diſtance from the globe or drop, the divergent light will be ſcarcely, if at all, perceptible; but if the globe be ſo ſituated, that thoſe rays which emerge parallel to each other, or at the greateſt poſſible angle with the incident, may arrive at the eye of the ſpectator, he will, by means of thoſe rays, behold it nearly with the ſame ſplendor at any diſtance.

N In like manner, thoſe rays which fall parallel on a globe, and are emitted after two reflections, ſuppoſe at the points F and G, will emerge, at H, parallel to each other, when the angle they make with the incident, A N, is the leaſt poſſible; and the globe muſt be ſeen very reſplendent, when its poſition is ſuch, that thoſe parallel rays fall on the eye of the ſpectator.

O The quantities of theſe angles are determined by calculation, the proportion of the ſines of incidence and refraction to each other being known. And this proportion being different (264, G) in rays which produce different colours, the angles muſt vary in each. Thus it is found, that the greateſt angle in rain-water for the leaſt refrangible, or red rays, emitted parallel after one reflection is 42° 2′, and for the moſt refrangible or violet rays emitted parallel after one reflection 40° 17′; likewiſe, after two reflections the leaſt refrangible or red rays will be emitted nearly parallel under an angle of 50° 57′, and the moſt refrangible or violet under an

Com

ariou

UNIV. OF MICH.

an angle of 54° 7′; and the intermediate colours will be emitted nearly parallel at intermediate angles.

Suppoſe now, that O (fig. 64.) is the ſpectator's eye, and O P a line drawn parallel to the Sun's rays, and let P O E, P O F, P O G, P O H, be angles of 40° 17′, 42° 2′, 50°, 57′, and 54° 7′ reſpectively, and theſe angles turned about their common ſide O P, will, with their other ſides O E, O F; O G, O H deſcribe the verges of two rainbows as in the figure. For, if E, F, G, H be drops placed any where in the conical ſuperficies deſcribed by O E, O F, O G, O H, and be illuminated by the Sun's rays S E, S F, S G, S H; the angle S E O being equal to the angle P O E, or 40° 17′, will be the greateſt angle in which the moſt refrangible rays can, after one reflection, be refracted to the eye, and therefore all the drops in the line O E muſt ſend the moſt refrangible rays moſt copiouſly to the eye, and thereby ſtrike the ſenſe with the deepeſt violet colour in that region. And in like manner the angle S F O being equal to the angle P O F, or 42° 2′, will be the greateſt in which the leaſt refrangible rays after one reflection can emerge out of the drops, and therefore thoſe rays muſt come moſt copiouſly to the eye from the drops in the line O F, and ſtrike the ſenſe with the deepeſt red colour in that region. And, by the ſame argument, the rays which have the intermediate degrees of refrangibility will come moſt copiouſly from drops between E and F, and ſtrike the ſenſes with the intermediate colours in the

order

order which their degrees of refrangibility require; that is, in the progreſs from E to F, or from the inſide of the bow to the outſide, in this order, violet, indigo, blue, green, yellow, orange, red. But the violet, by the mixture of the white light of the clouds, will appear faint, and inclined to purple.

Again, the angle SGO being equal to the angle POG, or 50° 57′, will be the leaſt angle in which the leaſt refrangible rays can, after two reflections, emerge out of the drops, and therefore the leaſt refrangible rays muſt come moſt copiouſly to the eye from the drops in the line OG, and ſtrike the ſenſe with the deepeſt red in that region. And the angle SHO being equal to the angle POH, or 54° 7′, will be the leaſt angle in which the moſt refrangible rays, after two reflections, can emerge out of the drops, and therefore thoſe rays muſt come moſt copiouſly to the eye from the drops in the line OH, and ſtrike the ſenſe with the deepeſt violet in that region. And, by the ſame argument, the drops in the regions between G and H will ſtrike the ſenſe with the intermediate colours in the order which their degrees of refrangibility require; that is, in the progreſs from G to H, or from the inſide of the bow to the outſide in this order, red, orange, yellow, green, blue, indigo, and violet. And ſince the four lines OE, OF, OG, OH may be ſituated any where in the above-mentioned conical ſuperficies, what is ſaid of the drops and colours in theſe lines is to be underſtood of the drops and colours every where in thoſe ſuperficies.

Thus there will be made two bows of colours, R an interior and ſtronger, by one reflection in the drops, and an exterior and fainter by two; for the light becomes fainter by every reflection; and their colours will lie in a contrary order to each other, the red of both bows bordering upon the ſpace G F, which is between the bows. The breadth of the interior bow, E O F, meaſured croſs the colours, will be 1° 45′, and the breadth of the exterior, G O H, will be 3° 10′, and the diſtance between them G O F, will be 8° 55′, the greateſt ſemidiameter of the innermoſt, that is, the angle P O F, being 42° 2′, and the leaſt ſemidiameter of the outermoſt P O G being 50° 57′. Theſe are the meaſures of the bows, as they would be, were the Sun but a point; for, by the breadth of its body, the breadth of the bows will be increaſed, and their diſtance diminiſhed by half a degree, and ſo the breadth of the interior iris will be 2° 15′, that of the exterior 3° 40′, their diſtance 8° 25′; the greateſt ſemidiameter of the interior bow 42° 17′, and the leaſt of the exterior 50° 42′. And ſuch are the dimenſions of the bows in the heavens found to be very nearly, when their colours appear ſtrong and perfect.

The light which comes through drops of rain by S two refractions without any reflection ought to appear ſtrongeſt at the diſtance of about 26 degrees from the Sun, and to decay gradually both ways as the diſtance from the Sun increaſes and decreaſes. And the ſame is to be underſtood of light tranſmitted through ſpherical hail-ſtones. And if the hail be a little flatted, as it often is, the light tranſmitted may grow ſo ſtrong at a little leſs diſtance than that of 26 degrees, as to form a halo about the Sun and Moon; which halo, as often as the ſtones are duly figured, may be coloured, and then it muſt be red within, by the leaſt refrangible rays, and blue without, by the moſt refrangible ones.

T The light which paſſes through a drop of rain after two refractions, and three or more reflections, is ſcarce ſtrong enough to cauſe a ſenſible bow.

CHAP. VI.

OF THE SEPARATION OF THE ORIGINAL RAYS OF LIGHT BY REFLECTION OR TRANSMISSION, THAT DEPENDS ON THE THICKNESS OF THE MEDIUM UPON WHICH THEY ARE INCIDENT.

U THE original or component rays of light are ſeparable from each other, not only by refraction, or by varying the angle of incidence on a reflecting ſurface, but are likewiſe at like incidences more or leſs reflexible, according to the thickneſs or diſtance between the two ſurfaces of
V the medium on which they fall. They are alſo liable to be turned out of their direct courſe by approaching within a certain diſtance from a body, by which means a ſeparation enſues, the rays be-

ing more or leſs deflected as they differ in colour. Of theſe circumſtances it will be proper to give ſome account.

If a glaſs or lens, whoſe ſurface is convex, or a W
portion of a ſphere, be laid upon another plain glaſs, it is evident that it will reſt or touch at one particular point only; and therefore, that at all other places between the adjacent ſurfaces will be interpoſed a thin plate of air, the thickneſs of which will increaſe in a certain ratio, according to the diſtance from the point of contact; that is to ſay, in arcs whoſe verſed ſines are very ſmall, as the diameter of the ſphere is to the ſine of the arc, ſo is that ſine to the verſed ſine or thickneſs of the air at the diſtance meaſured by the ſine.

Light incident upon ſuch a plate of air is diſ- X
poſed to be tranſmitted or reflected according to its thickneſs: thus, at the center of contact, the light is tranſmitted, and a black circular ſpot appears; this ſpot is environed by a circle, the colours of which, reckoning from the internal part, are blue, white, yellow, red; then follows another circular ſeries, viz. violet, blue, green, yellow, red; then purple, blue, green, yellow, red; green, red; greeniſh blue, red; greeniſh blue, pale red; greeniſh blue, reddiſh white.

Theſe are the colours which appear by reflec- Y
tion: by the tranſmitted light the following ſeries are ſeen. At the center white, then yellowiſh red, black; violet, blue, white, yellow, red; violet, blue, green, yellow, red, &c. ſo that the tranſ-

mitted light at any thickneſs, inſtead of white, appears of the compounded colour which it ought to have after the ſubtraction of ſome of the conſtituent colours by reflection; after which ſeries the colours become too faint and dilute to be diſcerned.

Z It is obſervable, that the glaſſes will not come into contact without a conſiderable degree of preſſure.

A By admeaſurement it appears, that the rays of any particular colour are diſpoſed to be reflected when the thickneſſes of the plate of air are as the numbers 1. 3. 5. 7. 9. 11. &c. and that the ſame rays are diſpoſed to be tranſmitted at the intermediate thickneſſes which are as the numbers 0. 2. 4. 6. 8. 10. &c.

B The places of reflection or tranſmiſſion of the ſeveral colours in a ſeries are ſo near each other, that the colours dilute each other by mixture, whence the number of ſeries in the open day-light ſeldom exceeds ſeven or eight: but if the ſyſtem be viewed through a priſm, by which means the rings of various colours are ſeparated according to their refrangibilities, they may be ſeen on that ſide towards which the refraction is made, ſo numerous,

C that it is impoſſible to count them. Or, if in a dark chamber the Sun's light be ſeparated into its original rays by a priſm, and a ray of one uncompounded colour be received upon the two glaſſes heretofore deſcribed, the number of circles will become very numerous, and both the reflected and tranſmitted light will remain of the ſame colour as the original incident ray. In this experiment it

alſo is ſeen, that in any ſeries, the circles formed
by the leſs refrangible rays exceed in magnitude
thoſe which are formed by the more refrangible
rays, and conſequently that in any ſeries the more D
refrangible rays are reflected at leſs thickneſſes
than thoſe which are leſs refrangible.

If the light be incident obliquely, the rings of E
colours dilate and enlarge themſelves; whence it
follows, that the thickneſs required to reflect the
colours of any ſeries is different in different obliquities.

Water, applied to the edges of the glaſſes, is F
attracted between them, and filling all the intercedent
ſpace, becomes a thin plate of the ſame dimenſions
as that which before was conſtituted of
air. In this caſe the rings become much fainter,
but vary not in their ſpecies, and are contracted
in diameter nearly in the proportion of 7 to 8:
conſequently the intervals of the glaſſes at like circles
cauſed by theſe two mediums, water and air,
are as about 3 to 4; that is, nearly as the ſines
which meaſure the angles of incidence and refraction,
made at a common ſurface between them.
And hence it may be ſuſpected, that if any other G
medium, more or leſs denſe than water, be compreſſed
between the two glaſſes, their intervals at
the rings cauſed thereby will be to the intervals at
which ſimilar rings are cauſed by the interjacent
air, as the ſine which meaſures the refraction made
out of air into that medium is to the ſine of the
incidence on the common ſurface.

These

H These are ſome of the phenomena of light incident on mediums which are environed by mediums of greater denſity, as air or water compreſſed or included between plates of glaſs. The ſame appearances follow, though with ſome little variation, when the colorific medium is denſer than that in which it is incloſed.

I It is well known that bubbles blown in ſoap-water exhibit a great variety of colours; but as theſe colours are commonly too much agitated by the external air to admit of any certain obſervation, it is neceſſary that the bubble be covered with a clear glaſs; in which ſituation the following appearances enſue: the colours emerge from the vertex or top of the bubble, and as it grows thinner by the ſubſidence of the water, they dilate into circles or rings parallel to the horizon, which ſlowly deſcend and vaniſh ſucceſſively at the bottom. This emergence continues till the water at the vertex becomes too thin to reflect the light, at which time a circular ſpot of an intenſe blackneſs appears at the top, which ſlowly dilates ſometimes to three quarters of an inch in breadth before the bubble breaks. Reckoning from the black central ſpot, the reflected colours are the ſame in ſucceſſion and quality as thoſe produced by the aforementioned plate of air, and the appearance of the bubble, if viewed by tranſmitted light, is alſo ſimilar to that of the plate of air in like circumſtances.

If the colours be viewed with different obliquities, their place is changed, but not near ſo much as in the plate of air. K

The end of a ſmall glaſs tube or pipe being melted, by turning the flame of a candle or lamp upon it, by means of a blow-pipe, may be blown into a bubble of an extreme thickneſs. Such a bubble will exhibit colours of the ſame kind as the foregoing, but much more briſk and lively. From which, and the premiſed obſervations, it is concluded, that a denſer medium incloſed by one that is rarer exhibits more lively colours than thoſe which are produced by a rarer medium included in one that is more denſe. It is alſo obſervable, that the colours produced by reflection from, or tranſmiſſion through, denſe ſubſtances, are leſs ſubject to vary by change of the obliquity of the incident light than they are in ſubſtances that are more rare. L M N

By wetting very thin plates of Moſcovy glaſs, whoſe thinneſs occaſion the like colours to appear, the colours become more faint and languid, eſpecially if wetted on the ſurface oppoſite to the eye; but no variation of their ſpecies is produced: ſo that the thickneſs of any plate, requiſite to produce any colour, ſeems to depend only on the denſity of the plate, and not on that of the ambient medium: and hence, if the ſuſpicion formerly urged be true (283, c), may be known the thickneſs which thin plates of any tranſparent ſubſtance have O at the place where a given colour in any ſeries is produced. For,

As the ſine of the angle of incidence at the common ſurface
Is to the ſine of the angle of refraction out of the given medium into air,
So is the thickneſs of a plate of air which exhibits the given colour
To the thickneſs of the given plate.

As lenſes ground to a long radius are neceſſary to be uſed in theſe experiments, and ſuch are not very common, it may be an acceptable piece of information for the learner to know, that their place may be well ſupplied by two pieces of plate-glaſs, or even common glaſs. If theſe be previouſly wiped, and then rubbed together, they will ſoon adhere with a conſiderable degree of force, and exhibit various ranges of colours, much broader than thoſe obtained by lenſes. One of the moſt remarkable circumſtances attending this method of making the experiment is the facility with which the colours may be removed, or even made to diſappear by heats too low to ſeparate the glaſſes. It ſeems moſt probable, that the operation of heat conſiſts in augmenting the diſtance between the ſurfaces. A touch of the finger immediately cauſes the irregular rings of colours to contract towards their center in the part touched.

CHAP. VII.

GENERAL INFERENCES RESPECTING THE DISPOSITION TO BE REFLECTED OR TRANSMITTED, INTO WHICH THE RAYS OF LIGHT ARE PUT, BY THE ACTION THAT DEPENDS ON THE THICKNESS OF THE MEDIUM UPON WHICH THEY ARE INCIDENT.

THE experiments or obſervations in the laſt Q
chapter being maturely weighed and conſidered, indicate the following theorem or general propoſition; namely,

Every ray of light in its paſſage through any re- R
fracting ſurface is put into a certain tranſient conſtitution or ſtate, which in the progreſs of the ray returns at equal intervals, and diſpoſes the ray, at every return, to be eaſily tranſmitted through the next refracting ſurface, and, between the returns, to be eaſily reflected by it.

For, by thoſe obſervations it appears, that one S
and the ſame ſort of rays, at equal angles of incidence on any thin tranſparent plate, is alternately reflected and tranſmitted for many ſucceſſions; accordingly, as the thickneſs of the plate increaſes in arithmetical progreſſion of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, &c. ſo that if the firſt reflection, or that which makes the firſt or innermoſt ring of colours, be made at the thickneſs 1, the rays ſhall

be tranſmitted at the thickneſſes 0, 2, 4 6, 8, 10, 12, &c. and thereby make the central ſpot and rings of light which appear by tranſmiſſion, and be reflected at the thickneſſes 1, 3, 5, 7, 9, 11, &c. and thereby make the rings which appear by reflection. And this alternate reflection and tranſmiſſion continues for a great number of viciſſitudes, and by other obſervations, which for the ſake of brevity are omitted, for many thouſands, being propagated from one ſurface of a glaſs-plate to the other, though the thickneſs of the plate be a quarter of an inch or above: ſo that this alternation ſeems to be propagated from every refracting ſurface to all diſtances without end or limitation. And becauſe the ray is diſpoſed to reflection at the thickneſſes 1, 3, 5, 7, &c. and to tranſmiſſion at the thickneſſes 0, 2, 4, 6, 8, &c. for its tranſmiſſion through the firſt ſurface is at the diſtance 0, and it is tranſmitted through both together, if their diſtance be infinitely little, or much leſs than 1, the diſpoſition to be tranſmitted at the diſtances 2, 4, 6, 8, &c. is to be accounted a return of the ſame diſpoſition which the ray firſt had at the diſtance 0, that is, at its tranſmiſſion through the firſt refracting ſurface.

T This alternate reflection and tranſmiſſion depends on both the ſurfaces of every thin plate, becauſe it depends on their diſtance. For if either ſurface of a thin plate of Moſcovy-glaſs be wetted, the colours grow faint (285, 0): it muſt therefore depend upon both.

It is therefore performed at the ſecond ſurface; for if it were performed at the firſt, before the rays arrive at the ſecond, it would not depend on the ſecond. u

It is alſo influenced by ſome action or diſpoſition, propagated from the firſt to the ſecond, becauſe otherwiſe at the ſecond it would not depend upon the firſt. And this action or diſpoſition, in its propagation, intermits and returns by equal intervals, becauſe in all its progreſs it inclines the ray at one diſtance from the firſt ſurface to be reflected by the ſecond, at another to be tranſmitted by it, and that, by equal intervals, for innumerable viciſſitudes. v

The returns of the diſpoſition of any ray to be reflected are termed its fits of eaſy reflection, and thoſe of its diſpoſition to be tranſmitted its fits of eaſy tranſmiſſion; and the ſpace it paſſes through between every return, and the next return, the interval of its fits. w

Thus, let CDFE (fig. 83.) repreſent a tranſparent medium, ſuppoſe water, upon which the ray AB is incident at a point in the upper ſurface O, O. Draw the line I, I, and let the interval between it and O, O be every where equal to the diſtance between the two ſurfaces of the plate of water, deſcribed in the laſt chapter (283, F), when the firſt ring of colour is reflected. Then, if the inferior ſurface of the medium were at I, I, the ray would be reflected upon the ſame principle as the ring of colour, and therefore at I, I it is in a fit x of eaſy reflection. Draw the parallel 2, 2 at the ſame diſtance from 1, 1, and the diſtance between 0, 0, and 2, 2 will be that thickneſs at which in the before-mentioned plate the firſt ring of colour is tranſmitted: the ray would therefore be tranſmitted if the inferior ſurface were at 2, 2, and conſequently it is there in a fit of eaſy tranſmiſſion. At 3, 3 it is again in a fit of eaſy reflection, and by applying the ſame argument to the equidiſtant lines 4, 4; 5, 5; 6, 6; 7, 7; 8, 8; it will appear that the ray will be alternately diſpoſed to tranſmiſſion and reflection; and if the laſt parallel or the inferior ſurface be diſtant from the ſuperior ſurface 0, 0, by an even number of intervals, the ray will arrive there in a fit of eaſy tranſmiſſion and emerge; but if the number be odd, it will arrive in a fit of eaſy reflection, and return back into the medium. The diſtance between the lines 0, 0 and 2, 2; 2, 2 and 4, 4, &c. are the intervals of the fits of eaſy tranſmiſſion, and the diſtances between 1, 1 and 3, 3; 3, 3 and 5, 5, &c. are the intervals of the fits of eaſy reflection.

Y What kind of action or diſpoſition this may be, whether it conſiſt in a circulating or a vibrating motion of the ray or of the medium, or ſomething elſe, experiments are wanting to determine. But the facts are not the leſs true on account of our ignorance of the mode of their origin. That truly great man, to whoſe penetration and induſtry we are indebted for almoſt all the knowledge we have of the phyſical properties of light, has, with great modeſty, propoſed an hypotheſis for the ſolution of theſe appearances. It is not without its difficulties, and muſt therefore be received with the ſame caution as it was propoſed, till experiment ſhall either confirm it, or ſubſtitute another theory in its place.

2 Sir Iſaac Newton's Hypotheſis. It may be ſuppoſed, that as ſtones by falling into water put the water into an undulating motion, and all bodies by percuſſion excite vibrations in the air; ſo the rays of light, by impinging on any refracting or reflecting ſurface, excite vibrations in the refracting or reflecting medium or ſubſtance, and by exciting them, agitate the ſolid parts of the refracting or reflecting body, and by agitating them, cauſe the body to grow warm or hot; that the vibrations thus excited are propagated in the refracting or reflecting medium or ſubſtance much after the manner that vibrations are propagated in the air for cauſing ſound, and move faſter than the rays, ſo as to overtake them; and that when any ray is in that part of the vibration which conſpires with its motion, it eaſily breaks through a refracting ſurface, but when it is in the contrary part of the vibration which impedes its motion, it is eaſily reflected; and, by conſequence, that every ray is ſucceſſively diſpoſed to be eaſily reflected or eaſily tranſmitted by every vibration which overtakes it.

CHAP. VIII.

OF THE PERMANENT COLOURS OF NATURAL BODIES, AND THE ANALOGY BETWEEN THEM AND THE COLOURS OF THIN TRANSPARENT PLATES.

A IT has already been ſhewn (272, E), that the colours of natural bodies conſiſt in a diſpoſition to reflect the rays of one ſort of light more copiouſly than the reſt. But their conſtitution, whereby they reflect ſome rays more copiouſly than others, remains to be diſcloſed.

B Thoſe ſuperficies of tranſparent bodies reflect the greateſt quantity of light, which have the greateſt refracting power; that is, which intercede mediums that differ moſt in their refractive denſities. And in the confines of equally refracting mediums there is no reflection.

C The analogy between reflection and refraction will appear by conſidering that the moſt refractive mediums totally reflect the rays of light at leſs angles of incidence, as was before ſhewn (270, A). But the truth of the propoſition will further appear by obſerving, that in the common ſuperficies of two tranſparent mediums, the reflection is ſtronger or weaker, accordingly as the ſuperficies hath a greater or leſs refractive power. If any tranſparent ſolid be immerged in water, its reflection becomes much weaker than before, and ſtill weaker if immerged in a fluid whoſe refracting power is yet ſtronger than that of water. If water be diſtinguiſhed into two parts by an imaginary ſurface, the reflection in the confine of thoſe two parts is none at all. In the confine of water and ice it is very little; in that of water and oil ſomething greater; in that of water and ſal-gemm ſtill greater; and in that of water and glaſs, or cryſtal, or other denſer ſubſtances ſtill greater, accordingly as thoſe mediums differ more or leſs in their refractive powers. D The reaſon then why uniform pellucid mediums, as water, glaſs or cryſtal, have no ſenſible reflection, but at their external ſuperficies, where they are adjacent to other mediums of a different denſity, is that all their contiguous parts have one and the ſame degree of denſity.

E The leaſt parts of almoſt all natural bodies, are in ſome meaſure tranſparent: and the opacity of bodies ariſes from the multitude of reflections cauſed in their internal parts.

F This may be eaſily ſeen by viewing ſmall ſubſtances with the microſcope or magnifying glaſs, for they appear for the moſt part tranſparent. And it may alſo be tried by means of the light received through a hole into a dark chamber. For any ſubſtance, how opake ſoever, if it be reduced to a ſufficient thinneſs, and applied to the hole, will appear manifeſtly tranſparent. Only white metalline bodies muſt be excepted, which, by reaſon of their very great denſity, ſeem to reflect almoſt all the light incident on their firſt ſuperficies, unleſs by ſolution in menſtruums, they be reduced into very ſmall particles, and then they alſo become tranſparent.

G Between the parts of opake or coloured bodies are many ſpaces, either empty or repleniſhed with mediums of other denſities; as water between the tinging corpuſcles with which any liquor is impregnated, air between the aqueous globules that conſtitute clouds and miſts; and for the moſt part, ſpaces void both of air and water, but yet, perhaps, not void of all ſubſtance, between the parts of hard bodies.

A The truth of this is evinced by the two precedent propoſitions (F, G): for, by the ſecond, there are many reflections made by the internal parts of bodies, which would not happen if the parts of thoſe bodies were continued without any ſuch interſtices between them; becauſe reflections are only made in ſuperficies which intercede mediums of different denſities (293, D.)

B A yet farther proof that the opacity of bodies ariſes from this diſcontinuation of their parts may be had, by conſidering that opake ſubſtances become tranſparent, by filling their pores with any ſubſtance of an equal or nearly equal denſity with their parts. Thus, paper dipped in water or oil, the oculus mundi ſtone ſteeped in water, linen cloth oiled or varniſhed, and many other ſubſtances ſoaked in ſuch liquors as will intimately prevade their pores, become by that means more tranſparent than otherwiſe; ſo, on the contrary, the moſt tranſparent ſubſtances may, by evacuating their pores, or ſeparating their parts, be rendered ſufficiently opake, as ſalts, or wet paper, or the oculus mundi ſtone, by being dried; horn, by being ſcraped; glaſs, by being reduced to powder, or otherwiſe flawed; turpentine, by being ſtirred about with water till they mix imperfectly; and water, by being formed into many ſmall bubbles, either alone in the form of froth, or by ſhaking it together with oil of turpentine, or ſome other convenient liquor with which it will not perfectly incorporate.

The parts of bodies and their interſtices muſt not be leſs than ſome definite bigneſs, to render them opake and coloured. C

For the opakeſt bodies, if their parts be ſubtilely divided, as metals, by being diſſolved in acid menſtruums, &c. become perfectly tranſparent. And it may alſo be remembered, that the black ſpot near the point of contact of the two plates of glaſs being of ſome conſiderable breadth, tranſmitted the whole light where the glaſſes did not abſolutely touch (281, Y.) And the reflection at the thinneſt part of the ſoap-bubble was ſo inſenſible as to make that part appear intenſely black, by the want of reflected light (284, I.) D

On theſe grounds it is, that water, ſalt, glaſs, ſtones, and ſuch like ſubſtances, are tranſparent. For on many conſiderations they ſeem to be as full of pores or interſtices between their parts as other bodies E are, but yet their parts and interſtices to be too ſmall to cauſe reflection in their common ſurfaces.

F The tranſparent parts of bodies, according to their ſeveral ſizes, muſt reflect rays of one colour, and tranſmit thoſe of another, on the ſame ground that thin plates or bubbles do reflect or tranſmit thoſe rays. And this appears to be the ground of all their colours.

G For if a thin body or plate, which, being of an even thickneſs, appears all over of one uniform colour, ſhould be ſlit into threads, or broken into fragments of the ſame thickneſs with the plate; there is no reaſon why every thread or fragment ſhould not keep its colour, and by conſequence, why a heap of thoſe threads or fragments ſhould not conſtitute a maſs or powder of the ſame colour which the plate exhibited before it was broken. And the parts of all bodies being like ſo many fragments of a plate, muſt on the ſame grounds exhibit the ſame colours.

H Now, that they do ſo, will appear by the affinity of their properties. The finely coloured feathers of ſome birds, and particularly thoſe of peacocks tails, do in the very ſame part of the feather appear of ſeveral colours in ſeveral poſitions of the eye. Likewiſe the fine-ſpun webs of ſome ſpiders appear coloured; and the fibres of ſome ſilks, by varying the poſition of the eye, do vary their colours. Alſo the colours of ſilks, cloths, and other ſubſtances which liquids can eaſily penetrate, become more faint by being wetted, much after the manner of the plate of Moſcovy glaſs, and recover their vigour again by being dried.

The air reflects the blue rays moſt plentifully, and muſt therefore tranſmit the red, orange, and yellow more copiouſly than the other rays. If the light of the ſetting-ſun, by paſſing through a long tract of air, be diveſted of the more reflexible rays, the green, blue indigo, and violet, the remainder, which is tranſmitted, will illuminate the weſtern clouds with an orange colour; and as the Sun ſets more and more, the tract of air through which the rays muſt paſs becomes longer, the yellow and orange are reflected, and the clouds grow more deeply red, till at length the diſappearance of the Sun leaves them of a leaden hue by the reflection of the blue light from the air. A ſimilar change of colour is obſerved on the ſnowy tops of the Alps in Switzerland, and the ſame may be ſeen, though leſs ſtrongly, on the eaſtern and weſtern fronts of white buildings; St. Paul's Church at London is a good object of this kind, and is often at ſun-ſet tinged with a conſiderable degree of redneſs. The ſame cauſe likewiſe occaſions the Moon in an eclipſe to aſſume a ruddy colour by the light tranſmitted through the atmoſphere (156, N, O.) I

The parts of bodies, on which their colours depend, are denſer than the medium which pervades their interſtices. K

For if they were not, the variation of colour, ariſing from the various obliquities of the incident light, (283, E. 285, K) would compound a mixt and imperfect colour, and never ſo vivid as experience L evinces. But when the parts are much denſer than the ambient medium, this variation is not ſo conſiderable; and therefore, the rays which are reflected leaſt obliquely may predominate over the reſt, ſo much as to cauſe a heap of ſuch particles to appear very intenſely of their colour.

M And hence the magnitude of the component parts of natural bodies may be conjectured by their colours.

N For, ſince the parts of theſe bodies are of about the ſame denſity as water or glaſs, as by many circumſtances is obvious to collect, it is highly probable that they exhibit the ſame colours with a plate of equal thickneſs. That colour being known, the thickneſs may be eaſily found by the preceding obſervations.

CHAP. IX.

OF THE INFLECTIONS OF THE RAYS OF LIGHT WHICH PASS IN THE VICINITIES OF BODIES.

O IT is obſervable, that if a beam of the Sun's light be let into a dark room through a very ſmall hole, the ſhadows of things in this light will be larger than they ought to be if the rays went on by the bodies in ſtrait lines, and that theſe ſhadows have three parallel fringes, bands, or ranks of colours adjacent to them. The principal circumſtances of the phenomenon are as follow:

P If a beam of the Sun's light be admitted into a darkened chamber through a hole of the breadth of the forty-ſecond part of an inch, or thereabouts, the ſhadows of hairs, thread, ſtraws, and other ſmall bodies, appear conſiderably broader than they would be if the light paſſed by them in ſtrait lines. For example; a hair, whoſe breadth was the 280th part of an inch, being held in this light at the diſtance of about twelve feet from the hole, did caſt a ſhadow which, at the diſtance of four inches from the hair, was the ſixtieth part of an inch broad, that is, above four times broader than the hair; and at the diſtance of ten feet, was the eighth part of an inch broad, that is, thirty-five times broader.

Nor is the effect altered by an alteration in the Q
denſity of the medium contiguous to the hair, for its ſhadow at like diſtances was equal, whether it was in the open air, or incloſed between two plates of wet glaſs, care being had that the incidence and emergence of the ray was perpendicular to the glaſſes. Scratches on the ſurface or veins in the body of poliſhed glaſſes did alſo caſt the like broad ſhadows. And therefore the great breadth of theſe ſhadows muſt proceed from ſome other cauſe than the uſual refraction which might ariſe from any action of the ambient medium.

Let the circle X (fig. 84.) repreſent the middle R
of the hair; ADG, BEH, CFI, three rays paſſing by one ſide of the hair at ſeveral diſtances; KNQ, LOR, MPS, three other rays paſſing by the other ſide of the hair at the like diſtances; D, E, F, and N, O, P, the places where the rays are bent in their paſſage by the hair; G, H, I, and Q, R, S, the

the places where the rays fall on a paper, G Q; I S the breadth of the ſhadow of the hair caſt on the paper; and T I, V S, two rays which fall on the points I and S, without being at all deflected by the action of the hair. Then it is manifeſt, that all the rays between T I and V S are bent in paſſing by the hair, and turned aſide from the ſhadow I S, becauſe, if any part of the light were not bent it would fall within the ſhadow, and there illuminate the paper, contrary to experience. And becauſe, when the paper is at a great diſtance from the hair, the ſhadow is broad, and therefore the rays T I and V S are at a great diſtance from each other, it follows that the hair acts upon the rays of light at a conſiderable diſtance in their paſſing by it. But becauſe the ſhadow of the hair is much broader in proportion to the diſtance of the paper from the hair when the paper is nearer to the hair than when it is at a great diſtance from it, it is evident that the action is ſtronger on the rays which paſs by at leaſt diſtances, and grows weaker and weaker accordingly as the rays paſs by at diſtances greater and greater, as is repreſented in the ſcheme.

The ſhadows of all bodies in this light are bordered with three parallel fringes or bands of coloured light, of which that contiguous to the ſhadow is broadeſt and moſt luminous, and that moſt remote from it is narroweſt, and ſo faint as ſcarcely to be viſible. If the light be received very obliquely on paper, or any other ſmooth white body, the colours may be plainly diſtinguiſhed in this order, viz. the

firſt

UNIV.

firſt or innermoſt fringe is violet, and deep blue next the ſhadow, and then light blue, green and yellow in the middle, and red without. The ſecond fringe is almoſt contiguous to the firſt, and the third to the ſecond, and both are blue within, and yellow and red without, but their colours are very faint, eſpecially thoſe of the third. The colours therefore proceed in this order from the ſhadow, violet, indigo, pale blue, green, yellow, red; blue, yellow, red; pale blue, pale yellow, and red.

If a larger beam of the Sun's light be admitted into a dark chamber, and part of it received on the blade of a ſharp knife, whoſe plane interſects the direction of the beam at right angles, while the other part is ſuffered to paſs by the edge of the knife, and received on a paper at the diſtance of about three feet; this laſt light will appear to ſhoot out or ſend forth two faint luminous ſtreams both ways into the ſhadow, ſomewhat like the tails of comets. Theſe ſtreams being very faint, are ſo much obſcured by the light of the principal direct rays, that it is neceſſary, in order to ſee them with any degree of diſtinctneſs, to let the direct rays paſs through a hole in the paper on to a piece of black cloth. The light of the ſtreams is then perceptible on the paper to the diſtance of ſix or eight inches from the Sun's direct light each way, and in all the progreſs from that direct light decreaſes gradually till it becomes inſenſible.

If two knife-blades, with ſtrait edges, be ſo fixed or ſet in a frame, that they may both be ſituated in the ſame plane, their edges parallel, and facing each other, and one of the blades moveable towards or from the other by means of a ſcrew, ſo that their parallelifm may be always preſerved, a beam of light may be ſuffered to paſs between their edges, and the appearances are the following: when the knives are at a conſiderable diſtance, ſo that the intromitted beam is broad, the ſtreams of light which ſhoot both ways into the ſhadow are ſcarce viſible, for the reaſon already mentioned, and the edges of the ſhadows are not bordered with coloured fringes, they becoming ſo broad that they run into each other, and by joining, form one continued light or whiteneſs at the beginning of the ſtreams. As the knives approach each other the fringes of colour appear on the confine of each ſhadow, becoming diſtincter and larger until they vaniſh, which happens when the edges are diſtant ſomewhat more than the 400th part of an inch. After the fringes have diſappeared, the line of light, which was in the middle between them, grows very broad, enlarging itſelf both ways into the ſtreams of light afore-mentioned; and when the knives are diſtant above the 400th part of an inch, the light parts in the middle, and leaves a ſhadow between the two parts. And as the knives ſtill approach each other, the ſhadow grows broader, and the ſtreams ſhorter at their inward ends, which are contiguous to the ſhadow, till upon the contact of the knives the whole light vaniſhes, leaving its place to the ſhadow *.

From theſe and ſome other experiments of the ſame tendency, it may be inferred,

That all bodies act upon the particles of light v attracting them when within a certain diſtance, and at greater diſtances repelling them; for the two comet-like ſtreams ſeem to be produced, the one by an attractive power exerted, by which the light is thrown into the ſhadow of the knife, and the other by a repulſion, by which it is turned towards the contrary part or region.

That theſe actions are ſtronger on thoſe rays w which paſs nearer the body than on thoſe which paſs at greater diſtances: conſequently thoſe rays which were parallel before their arrival in the vicinity of the body being variouſly deflected, muſt, after paſſing, diverge from each other; and, at the limit or diſtance at which attraction ceaſes, and repulſion begins, there muſt be a place at which the paſſing rays being very little affected by the action of the body, will proceed parallel, as before their arrival in its vicinity.

That this limitation or diſtance may differ in x rays of different colours, and cauſe the appearance

---

* The experiments of Newton on the inflection of light are few, and imperfect. Thoſe who have followed him in this delicate and highly important department of Natural Philoſophy, have done little more than add ſome inſulated facts to thoſe obſerved by him. The law followed by the powers that inflect light, and the limits of its action, are yet unknown.

of fringes: for, if the limit be leſs in the violet rays than in the red rays, the parallel rays of the violet colour will form a fringe, which ſhall be nearer the ſhadow of the body than that which is formed by the parallel rays of the red colour: and ſo of the intermediate colours will be formed intermediate fringes. It muſt, however, be confeſſed, that this ſuppoſition does not account for the repetition of the ſame colour at different diſtances.

CHAP. X.

OF THE POWERS BY WHICH BODIES REFLECT OR REFRACT THE RAYS OF LIGHT.

1. THE reflection of light is not cauſed by its impinging or ſtriking on the ſolid parts of bodies.

2. This will appear by the following conſiderations. Firſt, That in the paſſage of light out of glaſs into air, there is a reflection as ſtrong as in its paſſage out of air into glaſs, or rather a little ſtronger, and by many degrees ſtronger than in its paſſage out of glaſs into water. And it ſeems not probable, that air ſhould have more reflecting parts than water or glaſs. But if that ſhould poſſibly be ſuppoſed, A yet it will avail nothing; for the reflection is as ſtrong, or ſtronger, when the air is drawn away from the glaſs, as when it is adjacent to it. Secondly, If light in its paſſage out of glaſs into air be incident more obliquely (270, A) than at an angle of 40 or

41 degrees, it is wholly reflected; if leſs obliquely, it is in a great meaſure tranſmitted. Now, it is not to be imagined that light, at one degree of obliquity, ſhould meet with pores enough in the air to tranſmit the greater part of it, and at another degree of obliquity, ſhould meet with nothing but parts to reflect it wholly; eſpecially conſidering that in its paſſage out of air into glaſs, how oblique ſoever be its incidence, it finds pores enough in the glaſs to tranſmit the greateſt part of it. If any one ſuppoſe that it is not reflected by the air, but by the outmoſt ſuperficial parts of the glaſs, there is ſtill the ſame difficulty: beſides, that ſuch a ſuppoſition is unintelligible, and will alſo appear to be falſe, by applying water behind ſome part of the glaſs inſtead of air. For ſo in a convenient obliquity of the rays, ſuppoſe of 45 or 46 degrees, at which they are all reflected where the air is adjacent to the glaſs, they ſhall be in great meaſure tranſmitted where the water is adjacent to it; which argues that their reflection depends on the conſtitution of the air and water behind the glaſs, and not in the ſtriking of the rays upon the parts of the glaſs. Thirdly, If the colours made by a priſm placed at the entrance of a beam of light into a darkened room be ſucceſſively caſt on a ſecond priſm (271, c) placed at a diſtance from the former, in ſuch manner that they are all alike incident upon it, the ſecond priſm may be ſo inclined to the incident rays, that thoſe which are of a blue colour ſhall be all reflected by it, and yet thoſe of a red colour pretty

copiouſly tranſmitted. Now, if the reflection be cauſed by the parts of air or glaſs, it may be demanded why, at the ſame obliquity of incidence, the blue ſhould wholly impinge on thoſe parts, ſo as to be all reflected, and yet the red find pores enough to be in great meaſure tranſmitted. Fourthly, Where two glaſſes touch one another there is no ſenſible reflection (281, Y) and yet no reaſon can be given why the rays ſhould not impinge on the parts of the glaſs as much when contiguous to other glaſs as when contiguous to air. Fifthly, When the top of a ſoap water bubble, by the continual ſubſiding and exhaling of the water, becomes very thin, there is ſuch a little and almoſt inſenſible quantity of light reflected from it, that it appears intenſely black (284, I); whereas, round about that black ſpot, where the water is thicker, the reflection is ſo ſtrong as to make the water ſeem very white. Nor is it only at the leaſt thickneſs of thin plates or bubbles, that there is no manifeſt reflection, but at many other thickneſſes continually greater and greater. For we have ſeen that the rays of the ſame colour are by turns tranſmitted at one thickneſs, and reflected at another thickneſs for an indeterminate number of ſucceſſions. And yet, in the ſuperficies of the thin body, where it is of any one thickneſs, there are as many parts for the rays to impinge on as where it is of any other thickneſs. Sixthly, If reflection were cauſed by the parts of reflecting bodies, it would be impoſſible for thin plates or bubbles at the ſame place to reflect the rays of one colour, and tranſmit thoſe of another. For it is not to be imagined, that at one place the rays which, for inſtance, exhibit a blue colour, ſhould accidentally ſtrike upon the parts, and thoſe which exhibit a red upon the pores, of the body; and then at another place, where the body is either a little thicker or a little thinner, that on the contrary, the blue ſhould hit upon its pores, and the red upon its parts. Laſtly, Were the rays of light reflected by impinging on the ſolid parts of bodies, their reflectons from poliſhed bodies could not be ſo regular as they are. For in poliſhing glaſs with ſand, putty, or tripoly, it is not to be imagined that thoſe ſubſtances can, by grating and fretting the glaſs, bring all its leaſt particles to an accurate poliſh, ſo that all their ſurfaces ſhall be truly plane or truly ſpherical, and look all the ſame way, ſo as together to compoſe one even ſurface. The ſmaller the particles of thoſe ſubſtances are, the ſmaller will be the ſcratches by which they continually fret and wear away the glaſs until it be poliſhed; but be they ever ſo ſmall, they can wear away the glaſs no otherwiſe than by grating and ſcratching it, and breaking the protuberances, and therefore poliſh it no otherwiſe than by bringing its roughneſs to a very fine grain, ſo that the ſcratches and frettings of the ſurface become too ſmall to be viſible. And therefore, if light were reflected by impinging upon the ſolid parts of the glaſs, it would be ſcattered as much and as irregularly by the moſt poliſhed glaſs as by the roughest. So that it remains a problem, how glaſs poliſhed by fretting ſubſtances can reflect light ſo regularly as it does. And this problem is ſcarce otherwiſe to be ſolved than by ſaying, that the reflection of a ray is effected not by a ſingle point of the reflecting body, but by ſome power of the body which is evenly diffuſed all over its ſurface, and by which it acts upon the ray without immediate contact: for that the parts of bodies do act upon light at a diſtance, has already been ſhewn (301, T, U).

D Now, if light be reflected, not by impinging on the ſolid parts of bodies, but by ſome other principle, it is probable that as many of its rays as impinge on the ſolid parts of bodies are not reflected, but ſtifled or loſt in the bodies. For otherwiſe, we muſt allow two ſorts of reflections. Should all the rays be reflected which impinge on the ſolid parts of clear water or cryſtal, thoſe ſubſtances would rather have a cloudy colour than a clear tranſparency. To make bodies look black in all poſitions, it is neceſſary that many rays be ſtopped, retained, and loſt in them; and it is difficult to conceive that any rays can be ſtopt and ſtifled in them which do not impinge on their parts.

E Bodies reflect and refract light by one and the ſame power, variouſly exerciſed in various circumſtances.

This appears by ſeveral conſiderations. First, Becauſe when light goes out of glaſs into air as obliquely as it can poſſibly do, if its incidence be made ſtill more oblique, it becomes totally reflected (279, A.) For the power of the glaſs, after it has refracted the light as obliquely as is poſſible, if the incidence be ſtill made more oblique, becomes too ſtrong to let any of its rays go through, and by conſequence cauſes total reflection. Secondly, Becauſe light is alternately reflected and tranſmitted by thin plates of glaſs for many ſucceſſions (285, L) accordingly as the thickneſs of the plate increaſes in an arithmetical progreſſion. For here the thickneſs of the glaſs determines whether that power by which glaſs acts upon light ſhall cauſe it to be reflected, or ſuffer it to be tranſmitted. And thirdly, Becauſe thoſe ſurfaces of tranſparent bodies which have the greateſt refracting power do alſo reflect the greateſt quantity of light (292, B, C.)

The power by which bodies reflect and refract light, is the ſame as was ſhewn to be common to all bodies, and the cauſe of the inflection of the rays of light paſſing in their vicinities (303, U). For we muſt admit no more cauſes than are true, and ſufficient to explain the phenomena (6, I.) Such a cauſe is this; its exiſtence being proved, and its adequacy to the explanation of the reflection and refraction of light eaſy to be ſhewn. r

Let C D, (fig. 85.) repreſent the ſurface of a tranſparent body A, contiguous either to a vacuum B, or other medium poſſeſſing a leſs power of reflecting or refracting the rays of light. Let E F repreſent an imaginary ſurface at ſuch a diſtance from G

C D as to be ſituated at the limit of attraction, (303, W) that is to ſay, the ſpace between E F and C D is that in which, if a ray of light paſs, it will be attracted by the denſe body A, and on the other ſide towards B, near the line E F, a ray of light will be repelled.

H Suppoſe now G H to be a ray of light paſſing within the rare medium B, obliquely towards the ſurface C D, and let the line or part K H denote its velocity. This motion may (23, T) be reſolved into K I parallel, and I H perpendicular to C D. The attraction or repulſion exerted by the neareſt parts of the body A (and the other parts may be neglected) or by thoſe in the ſurface C D, muſt be aſſumed to act in the perpendicular to that ſurface, becauſe no reaſon can be given why it ſhould act towards one ſide more than another. It will therefore alter only the motion I H without affecting K I. When the light approaches E F it will be repelled; and if the force of repulſion in arriving at E F be greater than would generate the momentum I H, this laſt motion will be entirely deſtroyed before the light can arrive at the imaginary ſurface. The action of the repulſion, after it has deſtroyed I H, will, whatever may be its law, produce an equal velocity in the oppoſite direction. Conſequently the ray will deſcribe a motion compounded of H I and I L (equal to K I) and in the ſame direction; that is, it will paſs through the line H L, making the angle of reflection I H L equal to the angle of incidence I H K (262, Y).

Again, ſuppoſe M N to be a ray of light paſſing I within the rare medium B, which either by the more direct courſe towards the ſurface or otherwiſe has the perpendicular part O N of its motion too great to be deſtroyed by the repulſion experienced in approaching E F. It will paſs that imaginary ſurface, ſuffering only a diminution of its velocity eſtimated in the perpendicular O N. While it goes forward towards C D, its velocity in the perpendicular will be continually augmented by the attractive force; and if the whole accelerating force exceed the whole retarding force, as in this caſe experience ſhews it does, the light will enter and proceed in the denſe body with a velocity in the perpendicular Q S, greater than it had before in O N; the parallel velocity P O or S R ſtill continuing the ſame. The ray Q T will for this reaſon make a leſs angle S Q R with the perpendicular than before, inſtead of continuing in the line N U; that is, it will be refracted towards the perpendicular by entering the denſe body (262, A).

Again, ſuppoſe V W to be a ray of light paſſing K within the denſe body A, obliquely towards the ſurface C D. Reſolve the motion repreſented by Y W into Y X and X W, the firſt parallel, and the latter perpendicular to C D. The ray will paſs out of the denſe body into the ſpace between E F and C D; where, if the force of attraction towards C D on a ray during its paſſage to E F be greater than the momentum X W in the contrary direction, this laſt motion will be entirely deſtroyed before the light can arrive at the imaginary ſurface. Whence it follows, that for reaſons ſimilar to thoſe uſed in ſpeaking of the ray OH (310, H), the ray VW will be again returned towards CD, with a velocity equal and contrary to XW, which, together with XZ, the continuation of the uniform and unaltered velocity YX, will compound the actual motion WZ, making the angle of reflection XWZ equal to the angle of incidence XWY (262, Y).

L Laſtly, ſuppoſe TQ to be a ray of light paſſing within the denſe body A, which either by the more direct courſe towards the ſurface, or otherwiſe, has the perpendicular part SQ of its motion too great to be deſtroyed by the attraction experienced in its paſſage to EF. It will paſs that imaginary ſurface, ſuffering only a diminution of its velocity eſtimated in the perpendicular SQ. When it has gone beyond EF, its velocity in the perpendicular will be continually augmented by the repulſive force; and if the whole accelerating force be leſs than the whole retarding force, as in this caſe experience ſhews it is, the light will enter, and proceed in, the rare medium with a velocity in the perpendicular NO, leſs than it had before in SQ; the parallel velocity RS or OP ſtill continuing the ſame. The ray NP will for this reaſon make a greater angle ONP with the perpendicular than before; that is, it will be refracted from the perpendicular by entering the rare medium (262, Z, A).

M From theſe conſiderations it is deduced alſo, that the rays of light are not refracted or reflected all at once, but in refraction bent into a curve by the action of the body, ſo as to enter the ſurface of any medium more or leſs directly than they otherwiſe would have done, if its denſity had continued the ſame through the whole courſe of the rays. And in reflection, that the force acting in the direction of the perpendicular to the ſurface of a body, does not deſtroy the motion of the ray all at once, but bends it back in a curve. Which force, when it has deſtroyed that part of the motion of the ray which tended perpendicularly towards the common ſurface of the adjacent mediums, muſt reflect the ray with an equal angle and degree of velocity on the oppoſite ſide of the perpendicular to the point of incidence, or vertex of the curve. N This is evident from what has already been ſaid on the compoſition and reſolution of motion (23, T), and may, perhaps, without entering into particular explanations, be more readily conceived by attending to the motions of bodies projected obliquely from the Earth's ſurface; for here the aſcending or perpendicular part of the motion is gradually deſtroyed by the continually acting force, and a new, ſimilar, and equal motion is generated in the contrary direction, which, abſtracting the effect of the air's reſiſtance, cauſes the body to fall under an equal angle, and with the ſame velocity.

O If the forces of bodies upon the particles of light be ſuppoſed to act equally after the ratio of the maſſes of the particles, the rays will be all

equally refracted or reflected, however different
P their maſſes, provided their velocities be equal.
If the ſame law of the forces be ſuppoſed, and the
velocities of the particles be various, thoſe which
move with leſs velocities will ſuffer a greater de-
flection than thoſe which move with greater velo-
cities. The varying refrangibility and reflexibility
of the rays of light muſt ariſe either from the va-
rious velocities of the particles themſelves, or from
the action of bodies on the particles being ſtronger
on ſome than on others, after the ratio of their
Q maſſes. If the various velocities were the cauſe, the
moons of Jupiter, after being eclipſed, ought to
appear illuminated with a variety of colours, in
ſucceſſion, as the velocities of their conſtituent
rays cauſed them reſpectively to arrive at the eye
of the obſerver: and when light is diſperſed, by
refraction, into its component colours, the quan-
tity of this diſperſion ought in every medium
to be equal at equal mean refractions of the
whole ray: both which are contrary to experience.
R Whence it follows, in order to produce the va-
riety of refraction or reflection which happens in
the ſeveral rays of light, bodies muſt act on ſome
of the particles of light more ſtrongly than upon
others, after the ratio of their maſſes.

BOOK II.

SECT. II.

Of Optics.

CHAP. I.

CONCERNING THE REFLECTION AND REFRACTION OF LIGHT BY SURFACES REGULARLY FORMED.

BEFORE the diſcoveries of Sir Iſaac Newton A had ſhewn the compoſition of white light, the ſcience of optics conſiſted of propoſitions in which the rays of light were always ſuppoſed to be equally refrangible or reflexible. And, indeed, though the diſperſion of light, when refracted into its component colours, is the greateſt obſtacle to the perfection of the inſtruments now made; yet on moſt occaſions, with reſpect to viſion, we may regard a ray of white light as ſtill continuing white, even after refraction. For the colours of the ſpectrum into which it is dilated, are ſo near each other, when the incidence is near the perpendicular, that to ſenſe they form a white very little differing from that of the incident ray. But in ſtrict- B neſs, the general principles of optics are true only of any ſingle kind of rays.

C That bodies are viſible only by means of the light which they emit or reflect, is too evident to need any particular proof; and that every point of an illuminated ſurface emits the rays of light in all directions, is clear from the viſibility of the ſurface, to an eye in any poſition whatſoever: for if any part or ſenſible point of the ſurface did not emit light in a ſuppoſed or given direction, that point, to an eye placed in that direction, muſt be inviſible. But this effect never happens.

D The rays which proceed from a point are neceſ-
ſarily divergent, but if they fall on a reflecting
or refracting ſurface, they will be ſcattered in ſuch
directions as the conſtruction of the ſurface pro-
E duces. If the ſurface be properly formed, the
whole beam of rays may proceed, after reflection
or refraction, either diverging from ſome other
point, or parallel, or converging to a point.

F When the rays which are emitted or proceed
from any point are conſidered, that point is called
G the radiant point; when the rays which proceed to
any point are conſidered, that point is called the
H focus; and when the rays which proceed from a
whole ſurface or object, are conſidered, the body
of rays which is emitted from any one point, or
as much of it as is applied to uſe, is called a
pencil of rays.

I Since a pencil of rays, emanating from any given point of ſpace, is the means by which the ſight aſſures us, that a body exiſts at or in that point, it is plain that we are liable to deception in that reſpect: for if the pencil be ſo affected, either by reflection or refraction, as to proceed with a different divergency or direction, that is, in the ſame manner as it would have proceeded if emitted from ſome other point of ſpace, the ſenſe will refer the place of the object to the point which is in the direction of the laſt courſe of the rays (263, e).

Thus, if M R (fig. 86.) repreſent the ſection of a plane mirror, and O R an object, then the pencils O C and R D being reflected at C and D, will proceed to the eye at E, in the ſame manner as if emitted from points ſituated at I and M, and the ſame happening to the pencils which are emitted from the intermediate points between O and R, the ſenſe will refer the place of the object to I M. The ſame happens by refraction, as is clear from the conſideration of fig. 73. (263, E). P

If a pencil of rays be rendered convergent, ſo as to meet and croſs each other in a point, they will afterwards diverge, and the ſenſe will refer the place of the radiant point or object to the focus of the convergent rays, from which the divergence was laſt made; and, that rays of any ſort may be rendered thus convergent, either by reflection or refraction, is eaſily ſhewn. L

Suppoſe R (fig. 87.) to be a point, in any illuminated or luminous object, which emits a pencil conſiſting of ſeven rays of light, R A, R B, R C, M

RD, RE, RF, RG; let the ray RA be received on a ſpeculum, ſo placed as to reflect it through the point S: let another ſpeculum be adapted to receive and reflect RB alſo through S; and, in like manner, let the other rays be reflected through the ſame point; and the point S will become a radiant point, by means of the divergent rays, and will affect the ſenſe in the ſame manner as if the rays actually flowed from a body placed there. If the ſpeculums be ſuppoſed to touch each other, they will form a polygonal concavity. Suppoſe now the number of rays, inſtead of ſeven, to be infinite; then the adapted reflecting ſurface AG, inſtead of polygonal, muſt become curve, by reaſon of the infinite number of ſides. The ſame reaſoning may be applied to rays, which, inſtead of being emitted from a point, or diverging, fall on the reflecting ſurface, either converging to a point, or parallel to each other. It is therefore poſſible to conſtruct a ſuperficies that ſhall reflect into a focus the rays of light, which, either by converging or diverging, are directed either to or from any particular point.

Upon the ſame principles may be conſtructed ſpeculums, which ſhall cauſe the rays, after reflection, to diverge from any given point behind the reflecting ſurface. Thoſe ſpeculums, which cauſe the rays to become more divergent muſt be convex, and thoſe which cauſe them to become more convergent muſt be concave, as may eaſily be imagined.

The celebrated Archimedes, at the ſiege of Syracuſe, is ſaid to have deſtroyed the ſhips of Marcellus, by a machine compoſed of ſpeculums. P
Since a plane ſpeculum, in theory, reflects all the light which is incident upon it, under the ſame affections with which it was incident; the rays of the Sun, which, as coming from a vaſtly diſtant object, may be eſteemed parallel, will be reflected parallel to each other; and conſequently will heat and illuminate any ſubſtance on which they fall after reflection, in the ſame manner as if the Sun ſhone directly upon it. Two ſpeculums, which reflect the Sun's light on the ſame ſubſtance, will heat it twice as much as the Sun's direct light. Three will, in like circumſtances, heat it three times as much. And, by increaſing the number of ſpeculums, a prodigious degree of heat may be produced; more than ſufficient to conſume and deſtroy any inflammable ſubſtance.

Though a plane ſpeculum in theory is ſuppoſed to reflect all the light which falls upon it, yet in practice almoſt half the light is loſt, on account of the inaccuracy of the poliſh, and the want of perfect opacity in the ſubſtance of the mirror; on which accounts it happens that a conſiderable part of the light is ſcattered in all directions, and another part is abſorbed by the body. Q
The indefatigable Buffon, in the year 1747, was the firſt of the moderns who conſtructed a burning machine of this kind. It conſiſted of 168 quickſilvered glaſſes or ſpecula, each 8 inches long and 6 broad, R

6 broad, ſo contrived, that the focal diſtance might be varied, and alſo the number of glaſſes, as occaſions required. In the month of March, 1747, with 40 glaſſes he burnt a plank, at the diſtance of about 70 feet.

S If a body of rays, which either proceeds parallel, or, by converging or diverging, reſpects a given point, fall on the intercedent ſurface of two mediums of different refracting powers, the rays may be ſo refracted, if the ſurface be rightly formed, as to proceed parallel, or to converge to, or to diverge from, ſome other point.

T Let the polygonal ſurface A B C D E F G (fig. 88.) repreſent the ſurface intercedent between two mediums, the rarer being ſituated on the ſide towards R, and the denſer towards S; and let a pencil, compoſed of ſeven rays, R A, R B, R C, R D, R E, R F, R G, be incident, each ray on a different plane, as repreſented in the figure. Suppoſe the ray R A to be received on the ſurface at A, with an angle of incidence that correſponds to the angle of refraction which deflects the ray to the point S. And ſuppoſe the ray A B to be received leſs obliquely, or at a certain leſs angle of incidence; its angle of refraction will alſo be leſs, and it will proceed to S. And let a ſimilar adjuſtment of the planes at C, D, &c. be ſuppoſed, and the other rays will be refracted to the ſame point. Or if S be ſuppoſed the radiant point, the mediums being as before, U the focus will be at R. It is therefore plain, that rays proceeding out of a rare into a denſe medium, are rendered more convergent by a convex ſurface, and rays, proceeding out of a denſe into a rare medium, are rendered more convergent by a concave ſurface; and the contrary. Let the pencil conſiſt of an infinite number of rays, and the polygonal ſurface, adapted to refract it to a point, will, by reaſon of the infinite number of its ſides, become a curve. The ſame argument may be applied to rays that are either convergent or parallel at their incidence on the refracting ſurface. Conſequently, the intercedent ſurface of two mediums may be ſo formed as to refract into a focus, or render parallel, or divergent thoſe rays, which, at their incidence are either parallel, or do, by converging or diverging, reſpect any particular point.

From the eſtabliſhed laws of reflection and refraction, it is not difficult to inveſtigate the nature of the curves, into which the before-mentioned ſurfaces ought to be formed. But as the errors that ariſe from the uſe of ſpherical ſurfaces are very ſmall, and may be remedied by other means, and the mechanical or practical conſtruction of the required curves is very difficult, the parts of optical inſtruments are commonly formed ſpherical.

CHAP. II.

OF DIOPTRICS; OR THE REGULAR REFRACTION OF LIGHT.

x GLASS being a medium denſer and more refracting than the air, is uſed to make the tranſparent parts of optical inſtruments which are conſtructed to act by the principle of refraction. A piece of glaſs properly figured for that purpoſe is called a lens, and is diſtinguiſhed by the nature of its ſurfaces: thus A (fig. 89.) is a plano-convex, B a double convex, C a plano-concave, D a double concave, and E a convex-concave.

y The two firſt lenſes, A and B, nearly reſemble each other in their properties; for they refract converging or parallel rays to a point or focus, and refract diverging rays, ſo as either to make them meet in a focus or proceed leſs divergent than be-
z fore. If AB (fig. 71.) repreſent a double convex lens, and R a radiant point, then the rays which fall on the lens will be refracted to F, if the lens
A be of the requiſite convexity. For the rays that fall on the convex ſurface ACD. are rendered more convergent, and are made to converge ſtill more by falling on the concave ſurface ADB (320, U). The two following lenſes, C and D (fig. 89.), are referred to one ſpecies, on account of the reſemblance of their properties; for they render the incident rays more divergent than before, and therefore

fore cauſe diverging or parallel rays to diverge from
an imaginary or virtual focus, and refract con-
verging rays, ſo as either to make them diverge
from an imaginary focus, or proceed leſs conver-
gent than before. If AB (fig. 91.) repreſent a B
double concave lens, and R a radiant point, then
the rays which fall on the lens will be rendered
more divergent, and will proceed as if they had
proceeded from the point F, which is called the
virtual focus. The fifth lens E reſembles A and C
B, if its convexity be deeper, or a portion of a
leſs ſphere than its concavity: but if the con-
cavity be deepeſt, its properties reſemble thoſe of
C and D.

In the four firſt lenſes, the changes made in the D
courſe of the rays are more conſiderable the more
the ſurfaces are curved; but in the laſt the changes
are more conſiderable, the more the curvities of the
two ſurfaces differ from each other.

A right line, as RF (fig. 90.) paſſing through E
the center of any lens, and perpendicular to
both its ſurfaces, is called the axis of the lens.
The focus of rays that reſpect the axis, either by
falling parallel to it, or diverging from or converg-
ing to a point ſituated in it, is found in the axis,
and is called the principal focus.

A right line drawn from the point of convergence F
or divergence of any pencil of rays incident on a
lens, through the center of the lens, will paſs
through the focus of that pencil, if the point of

convergence or divergence be not ſituated far from the axis.

G The rays of light which diverge from the focus after paſſing through a lens, will occaſion the ſenſe to refer to that point, as if occupied by a lucid object (316, I); the focus, therefore, may be ſaid to be the picture or image of the radiant point. And as a ſurface may be conceived to be compoſed of an indefinite number of radiant points, the like number of focal points will appear, and conſequently a ſurface will be formed that will be H the image of the radiant ſurface. Let O B (fig. 92.) repreſent an object, and L N a double convex lens; from O and B through C the center, draw the lines O C I and B C M, and the foci of the points O and B will be found at I and M in thoſe lines (323, F), more or leſs diſtant from C, as the curvity of the ſurfaces of the glaſs is leſs or greater. The foci of the radiant points ſituated between O and B will be found between I and M, by the ſame proceſs. Conſequently an image will be there formed, reſembling the object, from each point of which rays of light will diverge in the ſame manner as from a real object; and its poſition, by reaſon that the rays croſs at C, will be inverted, or contrary to I the object itſelf, as appears by the figure. And becauſe the triangles O C B and I C M are ſimilar, the linear magnitudes of the image and the object be to each other reſpectively as their diſtances from the lens; for,

As the ſide CO, or diſtance of the object from
the lens,
Is to the ſide OB, or length of the object,
So is the ſide CI, or diſtance of the image,
To the ſide IM, or length of the image.

Again, let OB (fig. 93.) repreſent an object, K
and LN a double concave lens; draw OC and BC,
and the virtual foci of the points O and B will be
found at I and M in thoſe lines (323, F), more or
leſs diſtant from C, as the curvity of the ſurfaces
of the glaſs is leſs or greater. The intermediate
points of the object will have their intermediate
foci between I and M, and the poſition of the image
will be erect as well as the object. And becauſe L
the triangles OCB and ICM are ſimilar, the linear
magnitudes of the object and image will be as
their diſtances from the lens.

Hence it may be eaſily conceived, how convex M
lenſes become burning-glaſſes. For, as the object
and image, if viewed from the center of the lens
ſubtend the ſame angle, and the Sun is ſeen under
an angle of about half a degree, we may readily
find the denſity of the rays which form its image
in the focus of any lens. For example, if a lens, N
four inches broad, collect the Sun's rays into a
focus, at the diſtance of one foot, or twelve inches,
the image will not be more than $\frac{1}{10}$ of an inch
broad. The ſurface of this little circle, therefore,
will be 1600 times leſs than the ſurface of
the lens, and conſequently the Sun's light muſt
be ſo many times denſer within that circle. No

wonder, then, that it burns with a degree of violence and ardor far exceeding that of any culinary fire.

If a paper or white ſubſtance be held in the focus of a convex lens, the ſeveral foci of the radiant points of objects ſituated on the other ſide of the lens will illuminate as many points on the paper; which illuminated points agreeing in relative ſituation, intenſity, and colour with thoſe of the objects themſelves, will depict an exact and lively perſpective view of the ſame, though by reaſon of the croſſing of the rays, it will be inverted. But this phenomenon is ſcarcely to be ſeen, if any light be permitted to fall on the paper beſides that which paſſes through the lens; for which purpoſe the lens may be fixed in the window-ſhutter of a darkened chamber, as we ſhall have occaſion to remark in future.

UNIV. OF MICH.

C H A P. III.

OF THE EYE; AND OF VISION.

P IF the conſtruction of the univerſe were not ſo evident a proof of the exiſtence of a ſupremely wiſe and benevolent Creator, as to render particular arguments unneceſſary, (162, A) the ſtructure of the eye might be offered as one, by no means of the leaſt. This inſtance, among numberleſs others, demonſtrating that the beſt performances of art are infinitely ſhort of thoſe which are continually produced by the divine mechanic.

Q Though the apparatus, by which the eye is preſerved and kept in a ſtate proper for the quick motion and accurate direction towards the object to be viewed, is well worth attention and remark; yet, as it does not immediately come under our notice as illuſtrative of the principles of optics, we ſhall conſider only the globe of the eye, or organ by which viſion is performed.

R The eye is compoſed of ſeveral tunics or integuments, one within the other, and is filled within with tranſparent humors of different refractive denſities. The external tunic called the ſclerotica, is white on the anterior part, except a circular portion immediately in front, which is tranſparent, and more convex than the reſt of the eye: this tranſparent part is called the cornea. Immediately adherent to the ſclerotica within, is the choroides, or uvea, which, at the circumference of the cornea, becomes the iris, being expanded over great part of its ſurface, though not contiguous to it. The iris is compoſed of two kinds of muſcular fibres; the one ſort tend like the radii of a circle towards its center, and the others form a number of concentric circles round the ſame center. The central part of the iris is perforated, and the orifice, which is called the pupil, is of no conſtant magnitude; for, when a very luminous object is viewed, the circular fibres of the iris contract, and diminiſh its orifice; and on the other hand, when objects are dark and obſcure, the radial fibres of the iris contract, and enlarge the pupil ſo as to admit a greater quantity of light into the eye. The iris is variouſly coloured in different perſons, but according to no certain rule: in general, they whoſe hair and complexion are light coloured, have the iris blue or grey; and on the contrary, thoſe whoſe hair and complexion are dark, have the iris of a deep brown. But what ſpecific difference this may occaſion in the ſenſe, or whether any at all, is not diſcoverable. Within the uvea is another membrane, which at the circumference of the cornea becomes fibrous, and is called the ligamentum ciliare. This ligament is attached to the circumference of a double convex lens, whoſe axis correſponds with the center of the pupil; and which, by means of the fibres, can be altered in a ſmall degree in poſition, and perhaps in figure. The lens is termed the cryſtalline humor; and is included in a very ſtrong and tranſparent membrane, called the arachnoides. Between the cryſtalline humor and the cornea is contained a clear tranſparent fluid, called the aqueous humor; and between the cryſtalline humor and the poſterior part or bottom of the eye is included another clear tranſparent fluid, which is termed the vitreous humor. The refractive denſity of the cryſtalline is greater than thoſe of the humors that ſurround it. On the ſide next to the noſe a nerve is inſerted in the bottom of each eye, about twenty-five degrees from the axis of the cryſtalline, which, after entering the eye, is ſpread into an exceeding fine coat of network, termed the retina. Laſtly; a very black mucus or ſlime is ſpread over all the internal parts of the eye, that are not tranſparent, except the anterior part of the iris, which, as before obſerved, is coloured.

In the figure, the three concentric circles A B C (fig. 94.) repreſent the coats of the eye. The external coat, or ſclerotica, is tranſparent, and more convex between A and B, A K B being the cornea. The ſecond tunic, or uvea, is fibrous between D and I, and between G and H, and is the recalled the iris; the hole I H is the pupil. The third coat becomes fibrous between D and E, and between G and F, being there called the ligamentum ciliare, and is attached to the circumference of the lens or cryſtalline humor E F. The cavity or chamber A E F B is filled with the aqueous humor, and the chamber D N G F E is filled with the vitreous humor. At N is inſerted the optic nerve, the expanſion of which, over the internal ſurface D N G, is the retina.

T The manner in which the eye acts upon the rays of light may be thus explained. Let O L repreſent an object, and ſuppoſe a pencil of light to proceed from O, and enter the eye; then, becauſe the cornea is a convex concave lens, whoſe convexity is greateſt, (323, c) the rays will be rendered more convergent in paſſing through it; and if the cryſtalline be properly formed, they will be refracted by it into a focus at C on the retina. The ſame will happen to the pencil which proceeds from L, whoſe focus will be M; and the foci of the intermediate points will be between M and C: conſequently an inverted picture or image will be formed on the retina, and ſenſation be produced by the action of the light on the expanſion of the optic nerve, which from thence is conveyed to the
U ſenſorium. And that the parts of the eye are adapted to produce ſuch an image, appears likewiſe from experiment: for if the tunica ſclerotica be carefully taken away from the back of the eye of any animal, the inverted picture of external objects may be ſeen on the thin membranes which remain. Neither is the inverſion of the image any obſtacle to the mind's conceiving that the object is erect; for a focus at M may be conſidered as the indication of the exiſtence of a radiant point at L, and a focus at C may indicate the exiſtence of a radiant point at O; and ſo of others, the mind contem-
V plating the object itſelf, and not the image; beſides which,

which, we have notions reſpecting poſition that are not derived from the ſight, whence we judge whether a wall is perpendicular or a plane level, &c. Theſe notions are derived from a perception of the direction in which gravity conſtantly acts; to which direction we always refer. Whence it happens, that though the poſition of the eye be ever ſo much changed, the idea of the poſition of objects in view remains unaltered. For example; if an w obſerver view an upright pole or ſtaff, the image of the pole on the retina will be in a line at right angles to the opening of the eyelids, provided he holds his head upright; but if he vary the poſition of his head, the image will be formed in a different poſition, and upon a different part of the retina: notwithſtanding which, he conſtantly imagines the pole to be erect and unaltered.

Becauſe the foci of rays that differ in divergence x are found at different diſtances from the lens, thoſe which diverge leſs coming to a focus ſooner than thoſe which diverge more, it is neceſſary that the eye ſhould be adapted ſo as to act upon the rays that arrive from points at various diſtances, and to bring them to a focus upon the retina. The natural ſtructure of the eye is ſuch, that parallel rays have their focus on the retina; and when the proximity of any object cauſes its rays to fall with a greater divergency, the pupil of the eye contracts and excludes the moſt divergent rays, at the ſame time that the cryſtalline is brought forward, and perhaps rendered more convex by means of the ligamentum ciliare, by which proviſions the focus y ſtill falls on, the retina. This adjuſtment of the eye to the diſtances of objects gives the reaſon why we cannot view a near and a diſtant object at the ſame time; for, if a hair be held at a few inches diſtance between the eye and a remote object, ſuppoſe a tree at half a mile diſtance, the tree will appear confuſed and indiſtinct when the attention is fixed on the hair, and the ſame will be the caſe with the hair when the attention is fixed on the diſtant tree.

z There are ſome eyes naturally ſo defective, that they cannot effect this adjuſtment. Thoſe which are replete with humors have the cornea and cryſtalline too convex, ſo that the pencils come to their foci before their arrival at the retina, where they fall in ſmall circular ſpaces inſtead of points, and by their interference render the image confuſed: on the other hand, if the humors be deficient in quantity, the cornea and cryſtalline are too flat, and the pencils of rays not being ſufficiently refracted, arrive at the retina before their union in their foci; whence ariſes the ſame confuſion in the image as in the former caſe. They whoſe eyes are imperfect in the firſt manner are called myopes, from their winking or cloſing their eyelids, but more commonly near-ſighted becauſe they ſee very near objects diſtinctly, the divergency of the rays cauſing their foci to fall on the retina. They whoſe eyes are too flat are called preſbytæ, becauſe the imperfection of the ſight of old men being occaſioned by a decay of the humors, is generally of this kind. Both theſe imperfections may in a great meaſure be remedied by the uſe of proper ſpectacles. Since the rays converge too ſoon in the eyes of myopes, it is plain that a concave lens interpoſed between the object and the eye will cauſe the rays to fall more divergent, and conſequently will prevent their converging to a focus before their arrival at the retina. And the rays may be made to converge ſooner in the eyes of preſbytæ, by means of convex ſpectacles, ſo that they, being already convergent when they enter the eye, will be ſufficiently refracted by the cornea and cryſtalline to have their focus on the retina, and cauſe diſtinct viſion.

Theſe imperfections are much more frequently the conſequences of habit than is generally imagined. Studious men are generally near-ſighted, whereas ſailors, ſportſmen, and others, who are uſed to fix their attention on remote objects, are more ſubject to the contrary defect. The eyes of old men have another defect, namely, rigidity, or a want of the power of adjuſtment, ſo that it often happens that they require concaves for diſtant and convex lenſes for near objects, being capable only of ſeeing objects diſtinctly with the naked eye that are at a moderate diſtance. Every one ſhould avoid the uſe of ſpectacles as much as poſſible. For, though they render viſion more diſtinct, yet, they never fail to increaſe the defect of the eye, ſo as in time to render it almoſt impoſſible to ſee without them with any degree of diſtinctneſs.

B The eyes of various animals are accommodated with great ſkill to the exigences of their ſituations. In fiſhes the cornea is almoſt flat, that it may be no obſtacle to their ſpeed in the water, but this is compenſated by the cryſtalline, which is ſpherical, and therefore adapted to perform the whole neceſſary refraction of the rays. And in cats and ſome other animals that prey in the dark, the pupil of the eye is ſo variable as to admit more than an hundred times the quantity of light at one time than another. The human eye admits more than ten times the quantity of light at one time than at another, and perhaps the differences may be much greater in very dark places; it is not impoſſible but that the iris may be then almoſt intirely drawn back, and the pupil expanded to the whole ſurface of the cornea.

CHAP. IV.

OF REFRACTING MICROSCOPES; OR THE DIOPTRIC INSTRUMENTS, BY MEANS OF WHICH SMALL AND NEAR OBJECTS ARE SEEN MAGNIFIED.

THE apparent magnitude of any object C is meaſured by the angle under which it is viewed by the eye; conſequently the apparent magnitudes of two or more objects may be the ſame, or may differ in any proportion, let their real magnitudes be what they will. Thus, the apparent magnitudes, of CD, FG, and HI (fig. 95.) are equal when viewed by the eye at E, becauſe they are ſeen under the ſame angle, though their real magnitudes are very different: and the apparent magnitude of AB is greater than thoſe of the former three, becauſe it ſubtends a greater angle, though its real magnitude is equal to that of CD, and leſs than thoſe of FG and HI.

The image of any object on the retina will be D greater or leſs in proportion to its apparent magnitude, and therefore the ſame object is ſeen more enlarged and diſtinct the nearer it is brought to the eye, provided its diſtance be ſufficiently great for the rays to fall nearly parallel on the pupil: at leſs diſtances it continues to be enlarged, but is confuſed. The leaſt diſtance is about ſix inches. The eye can juſt diſtinguiſh objects that ſubtend an angle of half a minute of a degree, in which caſe the image on the retina is leſs than the $\frac{1}{7200}$ part of an inch broad, and the object, ſuppoſing it ſix inches diſtant, about the $\frac{1}{1200}$ part of an inch broad. And all ſmaller objects are inviſible to the naked eye.

1 The inſtruments by which thoſe ſmaller objects are rendered viſible are called microſcopes, and are conſtructed in two different methods. The one is, by the interpoſition of a convex lens between the object and the eye, to render it diſtinct at a leſs diſtance than ſix inches, by which means its apparent magnitude increaſes as the diſtance is diminiſhed: and the other is, by placing the object ſo with reſpect to a convex lens that its focal image may be much greater than itſelf, and contemplating that image inſtead of the object. The firſt are called ſimple or ſingle microſcopes, and the latter compound or double.

Let EY (fig. 96.) repreſent the eye, and OB a ſmall object, ſituated very near, ſo that the angle of its apparent magnitude OCB may be large. Then its image on the retina IM will alſo be large; but becauſe the pencils of rays are too divergent to be collected into their foci on the retina, it will be very confuſed and indiſtinct. Let the convex lens RS (fig. 97.) be interpoſed, ſo that the diſtance between it and the object may be equal to the focal length at which parallel rays would unite, and the rays which diverge from the object and paſs through the lens will afterwards proceed, and conſequently enter the eye, parallel: they will therefore unite, and form a diſtinct image on the retina, and the object will be clearly ſeen, though if removed to the diſtance of ſix inches, its ſmallneſs would render it inviſible. And ſince the apparent magnitudes of objects that ſubtend ſmall angles are nearly in the inverſe proportion of their diſtances, if the real magnitudes be equal, the proportion in which the object is magnified will be as ſix inches to its diſtance from the eye. Whence it follows, that the moſt convex lenſes, having the ſhorteſt focal diſtance of parallel rays, muſt magnify the moſt; for they permit the object to approach nearer the eye than thoſe do which are flatter. When the lens is not held cloſe to the eye, the object is amplified ſomewhat more; becauſe the pencils, which paſs at a diſtance from the center of the lens, are refracted inwards toward the axis, and conſequently ſeem to come from points more remote from the center of the object, as may be ſeen in fig. 98. where the pencils which are emitted from o and B, are refracted inwards, and ſeem to come from the points i and m.

A drop of water is a microſcope of this kind, by reaſon of its convex ſurface; for, if a ſmall hole be made in a plate of metal, or other thin ſubſtance, and carefully filled with a drop of water, ſmall objects may be ſeen through it very diſtinct, and much magnified. But there are ſome difficulties in the management of theſe, which ſmall glaſſes are free from, and therefore they are not much uſed. In fact, cheapneſs is their principal recommendation.

K The compound microſcope, by means of which the image is contemplated inſtead of the object, is of two kinds, the ſolar and the common double microſcope.

L The ſolar microſcope is thus conſtructed: let A C (fig. 99.) repreſent the ſide of a darkened chamber, L N a convex lens, fixed oppoſite a perforation in A C, O B a ſmall object, and P Q a white ſcreen placed within the chamber oppoſite to the lens; then, if the object be placed at a due diſtance from the lens, the pencil of light which proceeds from the point O will converge to a focus on the ſcreen at I, and the pencil which proceeds from the point B will converge to a focus at M, and the intermediate points of the object will be depicted between I and M, forming a picture which will be as much larger than the object in proportion as the diſtance of the ſcreen exceeds that of the image from the lens (324, 1). This is the principle on which the inſtrument acts, but it is uſual to add other auxiliary parts as a lens or ſpeculum to illuminate the object by converging the Sun's light upon it, &c. which cannot, with ſufficient brevity, be here enlarged upon. The ſolar microſcope is by far the moſt pleaſing in its effects, and leaſt offenſive to the eyes of any in uſe.

M In the common double microſcrope the image is contemplated inſtead of the object, being viewed through a ſingle lens in the ſame manner as the object in a ſingle microſcope. Thus,

N Let L N (fig. 100.) repreſent a double convex lens, and O B a ſmall object, ſo applied, that the pencils of rays which emerge from it, and paſs through the lens, may converge to their reſpective foci, and form an inverted image at I M. This image will be as much larger than the object in proportion as its diſtance exceeds that of the object from the lens (324, I), and, if it be viewed through the lens F G, will again be magnified upon the principle of the ſingle microſcope (336, F) in proportion as its diſtance from the eye is leſs than ſix inches; the image formed by the firſt lens, which is called the object-glaſs, ſerving inſtead of an object for the ſecond, or eye-glaſs. But it is to be noted, that the image formed in the focus of a lens differs from the real object in a very eſſential particular; that is to ſay, the light being emitted from the object in every direction, renders it viſible to an eye placed in any poſition, but the points of the image formed by a lens or mirror emitting no more than a ſmall conical body of rays, which arrives from the glaſs, can be viſible only when the eye is ſituated within its confine. Thus, the pencil which is emitted from B in the object, and is made to converge by the lens to M, proceeds afterwards diverging towards H, and therefore never arrives at the lens F G, nor enters the eye at E. But the pencils that proceed from the points o and b will be received on the lens F G, and by it carried, parallel, to the eye; conſequently the correſpondent points of the image i and m will be viſible, and thoſe which are ſituate farther out towards I and M

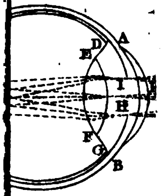
Q will not be ſeen. This quantity of the image I M, or viſible area, is called the field of view.

R Hence it appears, that if the image I M be large, a very ſmall part of it will be viſible, becauſe the pencils of rays will for the moſt part fall without the eye-glaſs F G. And it is likewiſe plain, that a remedy which would cauſe the pencils, that proceed from the extremes O and B of the object, to arrive at the eye will render a greater part of it viſible; or, in other words, enlarge the field view.

S This is effected by the interpoſition of a broad lens D E (fig. 101.) of a proper curvature at a ſmall diſtance from the focal image. For, by that means the pencil B M, which would otherwiſe have proceeded towards H, is refracted to the eye, as delineated in the figure, and the mind conceives from thence exiſtence of a radiant point at P, from which the rays laſt proceeded (316, 1). In the ſame manner, the other extreme of the image is ſeen at Q, and the intermediate points are alſo rendered
T viſible. On theſe conſiderations it is, that compound portable microſcopes are uſually made to conſiſt of an object lens, L N, by which the image is formed, enlarged, and inverted, an amplifying lens, D E, by which the field of view is enlarged, and an eye-glaſs or lens, by means of which the eye is allowed to approach very near, and conſequently to view the image under a very great angle of apparent magnitude *.

---

* The aberration of the refracted rays from the true focus, which ariſes from the ſpherical figure of the lens, D E, the priſmatic



Apparent Mag

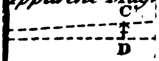
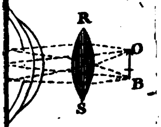
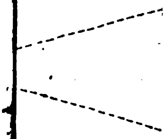


Fig. 97.



roscope. Fig. 99



UNI
OF
MIC

The magic lanthorn is a microſcope upon the ſame principles as the ſolar microſcope, and may be uſed with good effect for magnifying ſmall tranſparent objects; but in general it is applied to the purpoſe of amuſement, by caſting the ſpecies or image of a ſmall tranſparent painting on glaſs upon a white wall or ſcreen, at the focal diſtance from the inſtrument. After what has already been ſaid, it will be eaſy to underſtand the following deſcription of its component parts.

In the inſide of a box or lanthorn is placed the candle or lamp O (fig. 102,) whoſe light paſſes through the plano-convex lens N N, and ſtrongly illuminates the object O B, which is a tranſparent painting on glaſs, inverted and moveable before N N, by means of a ſliding piece in which the glaſs is ſet or fixed. This illumination is ſtill more increaſed by the reflection of light from a concave mirror, S S, placed at the other end of the box, that cauſes the light to fall upon the lens N N, as repreſented in the figure. Laſtly, a lens L L, fixed in a ſliding tube, is brought to the requiſite diſtance from the object O B, and a large erect image I M is formed upon the oppoſite wall,

---

matic colours that are ſeparated very much, and the loſs of light by reflection, which is moſt conſiderable when the refraction is greateſt, and the cauſes why in the beſt double microſcopes three or more lenſes are ſubſtituted inſtead of the ſingle amplifying lens, D E.

CHAP. V.

OF REFRACTING TELESCOPES; OR THE DIOPTRIC INSTRUMENTS, BY MEANS OF WHICH REMOTE OBJECTS ARE RENDERED LARGE AND DISTINCT TO THE VIEW.

w AS the microſcope is calculated to obviate the defects of viſion with regard to objects, whoſe angles of apparent magnitude are too ſmall for ſight on account of the ſmallneſs of the objects themſelves, ſo teleſcopes are adapted to improve the ſenſe with reſpect to objects, whoſe angles of apparent magnitude are too ſmall for ſight by reaſon of their remoteneſs or diſtance. The intention of both inſtruments is the ſame, namely, to increaſe that angle, and, by conſequence, the teleſcope differs very little from the compound microſcope, except in ſome particulars of convenience.

x Let L N (fig. 103.) repreſent a convex lens, and O B a diſtant object; then the pencils of rays will be collected into their reſpective foci, and form the inverted image I M, to which the eye, by means of the lens E E, may approach ſo near as to view it very large and diſtinct. This is the common aſtronomical teleſcope.

y But, as it is inconvenient to view objects on the earth inverted, there are uſually contrivances annexed to the teleſcope by which the image becomes erect as well as the object. The ſimpleſt of theſe is the following, where a concave is ſubſtituted inſtead of the convex eye-glaſs.

Let LN (fig. 104.) repreſent the object-glaſs as before, and OB a diſtant object. Then the pencils from the reſpective points of the object would converge to their foci, and form the inverted image IM, if the lens EE were not interpoſed. But the lens EE being a double concave, occaſions the rays to diverge more than before; ſo that the rays which are emitted from B in the object, inſtead of converging to M, are made to proceed parallel towards H. For the ſame reaſon the rays from O are made to proceed parallel towards K; the intermediate pencils being affected in the ſame manner. Now, ſince parallel rays cauſe diſtinct viſion, it is plain, that an eye placed in the pencil H, will conceive it to be emitted from ſome point, ſuppoſe m, ſituated in the laſt direction of the rays, and the image of B will be ſeen at m. By the ſame argument, the image of O will be ſeen at i, by an eye ſituated at K, and the like for the intermediate points. Therefore, an image will be ſeen at i m, erect or ſimilarly ſituated with the object itſelf. z

This teleſcope repreſents objects very bright and clear, and as much magnified as the other does, but is unpleaſant in its uſe, on account of the contracted field of view. For the pencils, being rendered divergent with reſpect to each other, paſs moſtly on one or the other ſide, without entering the pupil of the eye, and therefore a very ſmall A part of the image can be ſeen at once: thus, if the eye be at H, it will view the point m, and if it be moved towards K, it will ſee in ſucceſſion all the parts of the image towards i: but, as the pupil of the eye is not broad enough to receive the pencils H and K at the ſame time, the points m and i cannot be ſeen at once. The larger the pupil and the nearer it is placed to the eye-glaſs, the more pencils enter the eye at once. Conſequently the field of view is largeſt under theſe circumſtances, and in all other caſes leſs.

By the addition of two eye-glaſſes to the aſtronomical teleſcope, it is adapted to terreſtrial objects, the field of view remaining the ſame. Thus, the lens F F (fig. 105.) which is ſimilar to E E, being placed at twice the focal diſtance for parallel rays from E E, receives the pencils of parallel rays after they have croſſed each other at x, and forms an image at i m, ſimilar and equal to I M, but contrary in poſition, or erect; which laſt image is viewed by the lens G G. This is the common teleſcope, and though, by reaſon of the number of lenſes, it does not repreſent objects ſo bright as the foregoing, yet, its ample field of view makes it much more pleaſing and uſeful *.

---

* The eye piece of teleſcopes is uſually fitted up with five or more lenſes, for reaſons ſimilar to thoſe mentioned in the note on page 340. Their diſtances are often adjuſted in ſuch a manner that they magnify the firſt image I M on the compound microſcope principle.

The exhalations which continually riſe from the C
Earth, render the air leſs tranſparent, (293) eſpecially near the Earth, where the mixture is leſs complete, and therefore the celeſtial bodies are ſeen much more obſcure when in the horizon than when at any conſiderable elevation; for in the firſt caſe, they are viewed through that part of the atmoſphere which is contiguous to the ſurface of the Earth, and in the latter through a part which is at a greater diſtance. But this obſcurity is the leaſt part of the inconvenience. The riſing exha- D
lations have a kind of undulating motion, like that of ſmoke or ſteam, ſo that objects ſeen through them appear to have a tremulous or dancing motion, which is ſenſible even to the naked eye, if diſtant objects be viewed in a hot ſummer's day. Hence alſo the ſtars twinkle, and the ſhadow of lofty buildings have a tremulous motion. In te- E
leſcopes this effect is ſtill more perceptible, inſomuch as to render them intirely uſeleſs, for terreſtrial objects, when they augment the apparent magnitude more than eighty times *. But when objects in the heavens are viewed at any conſiderable altitude, inſtruments may be uſed which magnify many thouſands of times.

---

* That accurate and enlightened aſtronomer Alexander Aubert, Eſq. obſerves, that this undulation is the greateſt when the teleſcope is not placed in the open air, but within a room. For the temperature of the room being ſeldom correſpondent with that which obtains abroad, there is almoſt always a conſiderable undulation produced at the window where the ſtreams of hot and cold air mix. Herſchel uſes his teleſcopes in the open air.

F From this want of tranſparency in the atmoſphere ariſes that gradual diminution in the light of objects, which painters call the aerial perſpective; for, if the air were perfectly tranſparent, an object would be equally luminous at all diſtances, becauſe the viſible area and the denſity of light decreaſe in the ſame proportion, namely, as the ſquare
G of the diſtance. It is from this gradual diminution of light, together with the angle of apparent magnitude, that we eſtimate diſtances; and becauſe the celeſtial bodies, when near the horizon, are more obſcure, for the reaſon urged above (345, F), though their reſpective apparent magnitudes remain unaltered, or in a ſmall degree diminiſhed, we adopt the notion of their being actually larger at
H that time. Thus, likewiſe, men ſeen through a miſt appear gigantic, the obſcurity cauſing us to imagine them more diſtant than they really are.
I But, in the caſe of the heavenly bodies, there is another circumſtance that tends to deceive us in our judgment of the diſtance: we conceive the ſky to be a concave dome; and as the clouds towards the horizon are evidently more diſtant than thoſe near the zenith, we imagine the horizontal radius
K to be much longer than the vertical. From this notion we regulate our ideas with regard to the diſtance of the heavenly bodies, at different altitudes; which diſtance, we ſuppoſe to be greater than they are nearer the horizon, and we are conſequently led to imagine, that they are larger at that time.

By the ſolar microſcope and magic lanthorn, L we have ſeen that the ſpecies of near objects may be caſt on a ſcreen in a darkened chamber. The camera obſcura has the ſame relation to the teleſcope, as the ſolar microſcope has to the common double microſcope, and is thus conſtructed.

Let C D (fig. 106.) repreſent a darkened cham- M ber perforated at L, where a convex lens is fixed, the curvity of which is ſuch, that the focus of parallel rays falls upon the oppoſite wall. Then, if A B be an object at ſuch a diſtance, that the rays which proceed from any given point of its ſurface to the lens L, may be eſteemed parallel, an inverted picture will be formed on the oppoſite wall. For the pencil which proceeds from A will converge to a, and the pencil which proceeds from B, will converge to b, and the intermediate points of the object will be depicted between a and b. This is one of the moſt pleaſing and delightful experiments in optics, and never fails to ſtrike the beholder with ſurpriſe and admiration. Its only defect is the inverted poſition of the picture, which may be remedied by ſeveral methods. But as they all tend to make the image leſs lively, they are ſeldom uſed.

CHAP.

CHAP. VI.

OF THE IMPERFECTIONS OF TELESCOPES, AND THEIR REMEDIES; AND OF THE ACHROMATIC TELESCOPE.

N SINCE the conſtruction of a teleſcope conſiſts in nothing more than viewing, by means of a microſcope or eye-glaſs, the image which is formed in the focus of the object-glaſs; it may ſeem eaſy to make a teleſcope with a given object-glaſs, that ſhall magnify in any aſſignable degree. For, if the eye-glaſs be rendered more and more convex, the eye may be permitted to approach nearer and nearer to the image, and conſequently to view it under an angle of apparent magnitude that ſhall be greater and greater, as required. But this is unattainable on two ſeveral accounts. O The firſt is, that ſpherical ſurfaces do not refract the rays of light accurately to a point, as has already been obſerved; and the ſecond and moſt P conſequential is, that the rays of compounded light, being differently refrangible, come to their reſpective foci at different diſtances from the glaſs, the more refrangible rays converging ſooner than thoſe which are leſs refrangible. This is evident from what has already been ſaid on that ſubject, Q and is likewiſe confirmed by experiment; for a paper, painted intenſely red, and properly illuminated,

nated, will caſt its ſpecies, by means of a lens, on a ſcreen at a greater diſtance than will another blue paper by the ſame lens in like circumſtances. And here it may be noted, that the lens proper for this experiment muſt be very flat, or a portion of the ſurface of a large ſphere. Hence the ſpecies or image of a white object may be ſaid to conſiſt of an indefinite number of coloured images, the violet being neareſt, and the red fartheſt from the lens, and the images of intermediate colours at intermediate diſtances. The aggregate, or image itſelf, muſt therefore be in ſome degree confuſed, and this confuſion, being very much increaſed by the magnifying power, or eye-glaſs, renders it neceſſary to uſe an eye-glaſs of a certain limited convexity to a given object-glaſs. For which reaſon, if it be required to conſtruct a teleſcope that ſhall magnify objects in a greater degree than a given teleſcope, the object-glaſs muſt be leſs convex, and of conſequence its focal diſtance longer. Thus an object-glaſs of 4 feet focal length will bear an eye-glaſs of about 1⅕ inch focus, and will magnify objects in length or diameter 40 times: one of 25 feet focal length will bear an eye-glaſs of 3 inches focus, and magnifies 100 times; and one of 100 feet will bear an eye-glaſs of ſix inches, and magnifies 200 times. It is alſo neceſſary to limit the aperture of the object-glaſs, to exclude thoſe rays which are incident at too great diſtances from the center; for thoſe, being more refracted, are more particularly ſubject to the irregularities which ariſe, either from the figure of the glaſs or the unequal refraction of light. The diameter of the apertures of object lenſes, of equal goodneſs, ſhould be as the ſquare roots of their focal lengths.

v The great inconvenience and difficulty of managing
the longer teleſcopes, occaſioned the philoſophic
world to fix their thoughts upon the
means of converging the rays of light without ſeparating
them into their component colours. The
expedients for that purpoſe were firſt perfected by
w Sir Iſaac Newton and Mr. Dollond. The focal
image in the teleſcope of Sir Iſaac Newton is formed
by reflection from ſpeculums or mirrors, and being
therefore free from the irregular convergency
of the various rays of light, will admit of a much
larger aperture, and bear the application of a very
x great magnifying power. The difficulties which
attend this inſtrument, are the tarniſhing of the
metalline ſpeculums, and the very great accuracy
required in giving them the true figure, for an
error in a reflecting ſurface affects the direction
of the rays much more than a like error in a refracting
y ſurface. Yet this teleſcope is, notwithſtanding,
z the beſt in uſe. Mr. Dollond's invention
conſiſts in the uſe of a compound object-glaſs,
which is uſually termed achromatic, or colourleſs,
from its property; and the principal imperfection
in the practice, is the difficulty of procuring
glaſs that ſhall be uniformly of the ſame
refractive denſity. As we are now ſpeaking of
dioptrics,

dioptrics, it will be more regular to deſcribe the achromatic teleſcope firſt, and refer the other to its place, where we ſhall explain the properties of inſtruments that act on the principle of reflection.

Becauſe the component rays of light differ A
from each other in refrangibility, they are ſeparated
from each other by refraction, and becauſe
they are all refracted ſo as to preſerve a
conſtant ratio between the ſines of the angles of
incidence and refraction, that ſeparation muſt be
greateſt when the whole beam of light is moſt deflected
from its courſe. From hence opticians have
concluded, and there is a paſſage in Sir Iſaac
Newton's * optics, that ſeems to confirm the opinion,
that priſms, which deflect the whole beam
of light equally out of its courſe at like incidences,
will, however different their refractive denſities,
occaſion alſo an equal ſeparation or divergency of
the component rays: or, in other words, that if
the emergent refracted light from the ſurface of a
given priſm be immediately received on the ſurface
of a ſecond priſm, which ſhall refract it equally
in the contrary direction, ſo that at its emergence,
it ſhall proceed parallel to the firſt incident
beam, this laſt emergent light will continue white,
however different the matter of the ſecond priſm
may be from that of the firſt. But this Mr. Dollond B
has ſhewn to be ill-founded, for, by his experiments
it appears, that the different kinds of

---

* Book I. Part 2. Experiment VIII.

glaſs differ extremely with reſpect to the divergency of colours produced by equal refractions. He found that two priſms, one of white flint-glaſs, whoſe refracting angle was about 25 degrees, and another of crown-glaſs, whoſe refracting angle was about 29 degrees, refracted the beam of light nearly alike, but that the divergency of colour in the white-flint was conſiderably more than in the crown-glaſs; ſo that, when they were applied together, to refract contrary ways, and a beam of light tranſmitted through them, though the emergent continued parallel to the incident part, it was, notwithſtanding, ſeparated into component colours. Whence he inferred, that, in order to render the emergent beam white, it is neceſſary that the refracting angle of the priſm of crown-glaſs ſhould be increaſed; and by repeated experiments, he diſcovered the exact quantity. But this colourleſs emergent light was not then, by reaſon of the increaſed angle of the priſm of crown-glaſs, parallel to the incident ray, but was refracted towards the baſe of the laſt mentioned priſm.

By theſe means he obtained a theory, in which refraction was performed without any ſeparation or divergency of colour, and which it was not difficult to apply in the conſtruction of the object-glaſſes of teleſcopes. Let ABED (fig. 187.) repreſent a double concave lens of white flint-glaſs, and AGDF a double concave of crown-glaſs; then the parts of the lenſes which are on the ſame ſide of the common axis, namely, ACB and AFG, may be conceived to act like two priſms, which refract contrary ways, and if the exceſs of refraction in the crown-glaſs AFO be ſuch as preciſely to deſtroy the divergency of colour cauſed by the flint-glaſs ACB, the incident ray SH will be refracted to x, without any production of colour. The ſame is alſo true of the ray s h, and of all the other incident rays, and conſequently the whole focal image formed by this compound object-glaſs will be achromatic, or free from colour which might ariſe from refraction. It will therefore bear a larger aperture, and greater magnifying power, and of courſe enlarge objects much more than a common refracting teleſcope of the ſame length.

It is more convenient on ſeveral accounts to combine three lenſes together, one double concave of flint-glaſs between two convexes of different kinds of crown-glaſs; and the glaſſes may be ſo adjuſted to each other, as not only to form a focal image without the priſmatic colours, but alſo free from the defects which in other lenſes ariſe from their ſpherical figure. s

The greateſt impediment to the conſtruction of large achromatic teleſcopes, as has been obſerved, (350, z) is the want of a flint-glaſs of an uniform refracting denſity. Fortunately for Dollond, this kind of glaſs was procurable when he began to make achromatic teleſcopes, though the attempts r of many ingenious chemiſts have ſince been exerted to make it, without much ſucceſs *.

CHAP. VII.

OF CATOPTRICS, OR THE REGULAR REFLECTION OF LIGHT; AND OF THE REFLECTING TELESCOPE.

G IT has been ſhewn, (317, M) that a ſurface may be conſtructed that ſhall reflect the rays of a given pencil of light, ſo as to make them either converge to a point, diverge from a point, or proceed parallel to each other. This ſurface may be either plane or curved.

H A plane mirror reflects a pencil of light under the ſame circumſtances as it was incident; that is to ſay, if a pencil, which emanates from a given point, be incident on the mirror, it is reflected ſo, that its rays proceed with the ſame divergency from another point, whoſe diſtance behind the mirror is equal to the diſtance of the radiant point

---

* The author is credibly informed, that the glaſs employed by Dollond in the fabrication of his beſt teleſcopes, was all of the ſame melting, or made at one time; and that, excepting this particular treaſure, caſually obtained, good denſe glaſs for achromatic purpoſes was always as difficult to be procured as it is now.

UNIV. OF

before the mirror from the place of [illegible] and if the pencils of rays which [illegible] from a given ſurface, be [illegible] mirror, they will be reflected ſo as to [illegible] to each other as before [illegible] appear to proceed from [illegible] and diſtance [illegible] of the [illegible] that plane mirrors [illegible] are equal [illegible] objects themſelves.

Concave [illegible] which are [illegible] and convex [illegible] (318, o.) [illegible] formed, according to the [illegible] divergent [illegible] will reſpect ſome [illegible]

A portion [illegible] fifteen degrees [illegible] the curve ſurface, by wh[illegible] by reflection to [illegible] point. It is therefore [illegible] purpoſe, as being much eaſier to [illegible]

There is a great [illegible] the properties of convex [illegible] and concave mirrors, and between the properties of [illegible] and convex mirrors. Convex [illegible] and concave mirrors [illegible] form an inverted focal image of any remote object by the convergence of the pencils of rays [illegible] concave lenſes and convex [illegible] form an [illegible] before the mirror from the place of incidence: and if the pencils of rays, which emanate from a given ſurface, be incident on the mirror, they will be reflected ſo as to preſerve the ſame inclinations to each other as before, and therefore will appear to proceed from a ſurface, whoſe magnitude and diſtance behind the mirror are exactly equal to thoſe of the radiant ſurface (317, K.) Hence it is, that plane mirrors reflect the ſpecies of objects, which are equal, like, and ſimilar in poſition with the objects themſelves.

Concave mirrors render the pencils of rays, I
which are incident upon them, more convergent, and convex mirrors render them more divergent, (318, O.) If the mirrors be regularly formed, according to the proper curve, the convergent or divergent light of any pencil after reflection will reſpect ſome particular point of ſpace.

A portion of a ſphere, whole breadth is about K
fifteen degrees, differs very little from the curve ſurface, by which parallel rays would be made by reflection to converge to, or diverge from, a point. It is therefore in many caſes uſed for that purpoſe, as being much eaſier to conſtruct.

There is a great reſemblance between the pro- L
perties of convex lenſes and concave mirrors, and
between the properties of concave lenſes and con-
vex mirrors. Convex lenſes and concave mirrors M
form an inverted focal image of any remote object
by the convergence of the pencils of rays: concave N
lenſes and convex mirrors do, in general, form an

erect image in the virtual focus, by the divergence
O of the pencils of rays. In thoſe inſtruments,
whoſe performances are the effects of reflection,
the concave mirror is ſubſtituted in the
place of the convex lens, and the convex mirror
P may be uſed inſtead of the concave lens: but their
diſpoſitions with reſpect to each other, when combined,
muſt neceſſarily differ from thoſe of lenſes,
on account of the opacity of the one, and the tranſparency
of the other.

Q Let AR (fig. 108.) repreſent the poliſhed ſpherical ſurface of a concave mirror, and OB an object ſituated without the center of the mirror; then the pencil of rays, which is emitted from the point O, will fall on the mirror; and, after reflection, converge to the focus I; the pencil from B will converge to M, and the like will happen to thoſe emitted from the intermediate points, whoſe foci will be found between I and M. There will conſequently be formed before the mirror an inverted focal image, reſembling that which is formed by a convex lens (324, H.)

R Let AR (fig. 109.) repreſent the poliſhed ſpherical ſurface of a convex mirror, and OB an object: then the pencil of rays, which is emitted from the point O, will fall on the mirror, and after reflection diverge from the virtual focus I; the pencil from B will emerge from M, and the like will happen to thoſe emitted from the intermediate points, whoſe virtual foci will be found between I and M. There will conſequently be formed

formed behind the mirror an erect focal image, reſembling that which is formed by a concave lens.

Let AR (fig. 110.) repreſent a concave mirror, whoſe center is C, and OB an object ſituated without the center. Through the center C, from O, draw the line ON, which will be perpendicular to the mirror at N, and will therefore repreſent both the incident and reflected ray, which proceeds from O, and is reflected at N: the focal repreſentation or image of O will conſequently be found in that line. Through C, from B, draw the line BV, and by the ſame reaſoning the focal image of B will be found in that line. Draw the line or ray OV, and it will be reflected ſo as to croſs the ray ON at I, the angle of reflection IVC being equal to the angle of incidence OVC. This interſection of the rays determines the focal point of O, which is I. From B to N draw the ray BN, and its reflection will determine the focus of B, which is M, and the image will be inverted.

Let AR (fig. 111.) repreſent a convex mirror, and the other repreſentations and conſtruction be as in the laſt figure. The focal repreſentations of O and B will be found in the lines OC and BC, and the reflected part of the ray OV will virtually croſs the line OC at I; the reflected part of the ray BN will alſo virtually croſs the line BC at M. Theſe interſections will determine the place of the focal image IM, which will be erect.

Hence

u Hence it appears to be the property of these mirrors, that the object and the image, if viewed from the center of the ſphere, are ſeen under equal angles; for, the angle OCB is equal to the angle ICM; and that the object and image, if viewed from the point of reflection, are ſeen under equal angles; for, the angle OVB is equal to the angle IVM. From this it is eaſy to find the poſition and magnitude of the focal image, if the poſition and magnitude of the object, and the diameter of the ſphere, of which the mirror is a part, be known.

w The reflecting teleſcope, which was made by Sir Iſaac Newton, was of the following form.

Let DBFG (fig. 112.) repreſent a tube, at one end of which is placed the concave mirror AR, and let OB repreſent a diſtant object; then the pencils, which are emitted from the ſeveral points of its ſurface, will be collected, and form an inverted image i m. But by the interpoſition of the plane mirror KC, the rays are reflected, and the image is formed at IM, which is ſeen very much magnified by means of the plano-convex lens at L.

x The immenſely powerful teleſcopes of Herſchel are on this conſtruction. This capital artiſt, and moſt aſſiduous aſtronomer, has made ſeveral ſpeculums, which are ſo perfect as to bear a magnifying power of more than ſix thouſand times in diameter on ſome of the fixed ſtars *. The largeſt

---

* Phil. Tranſ. 1784.

teleſcope completed by him, and far exceeding in all reſpects any yet attempted, has an object ſpeculum of twenty feet focal length, and bears an aperture of eighteen inches and ſeven tenths of an inch!

The reflecting teleſcope, which is moſt in uſe at preſent, is compoſed of two concave mirrors of different radii. The larger concave AR (fig. 113) forms the focal image IM, which ſerves as an object for the ſmall mirror KC: a ſecond image I′M′ is formed by the mirror, the rays paſſing through the amplifying lens L, which is placed in a hole or perforation in the center of the great mirror AR. This image is erect, and is viewed much enlarged through the eye-glaſs or lens P. Y

In good reflecting teleſcopes the object ſpeculum is not of a ſpherical form. Z

Reflecting microſcopes are ſometimes made; the method of conſtructing which, as alſo of other inſtruments, may be deduced from what has been ſaid concerning reflecting teleſcopes. A

The end of the firſt volume.

INDEX

TO THE

FIRST VOLUME.

A.

ABERRATION, 254.
Acceleration, 27.
Achromatic teleſcope, 351.
Aerial perſpective, 345.
Air, its colour, 297. want of tranſparency, 345.
Algol, 192. 196.
Alps, their colour, 297.
Analogy, 2. 4.
Analyſis and ſyntheſis, 100.
Animalculæ, 12.
Aperture, 350.
Aphelium, 122.
Apogee, 122.
Appearances, *mutual*, of the planets, 113—115.
Apſides, 121. their motion, 232.
Archimedes, 319.
Attraction, 7. 8. 25. wherein it differs from impulſe, 33. of coheſion, 44. & ſeq. is very properly aſſumed as a general cauſe of phenomena, 50. apparently ſubſiſting between floating bodies, 48—50. See *Gravity*, *Coheſion*.
Atwood, Mr. his inſtrument for meaſuring the ſpaces deſcribed by bodies moving either uniformly or otherwiſe, 43.
Axiom, 2.
Axis and wheel, 60.

B

Balance, 59.
Block, 61.
Bradley, Dr. 240. 254.
Buffon, his burning machine, 319.
Burning mirrors, 319. glaſſes, 325.

C

Camera obſcura, 346.
Cauſe and effect, 4. 6. 50. 51.
Cauſe, firſt, is intelligent, 103.
Center, of gravity, 74—79.
Centripetal force, 94 & ſeq. 197 & ſeq.
Circumnavigation, 110.
Clocks, 88 & ſeq.
Clouds, why of ſuch various colours, 297. a deception ariſing from their poſition, 346.
Coheſion, 44 & ſeq.
Cold, 16.
Colour, wherein it conſiſts, 271. 272. 292 & ſeq.
Comets, 159—162. 167. their motion explained, 206.
Compoſition and reſolution of motion or force, 23. 24. 62. 69.
Copernicus, 103.
Creator, 50. 111.

D.

Dead beat eſcapement, 89.
Declination, 182.
Degree, meaſure of, 236.
Demonſtration, 2.
Dioptrics, 332. & ſeq.
Diviſibility of matter, 10—13.
Dollond, 350.

E

Earth, proofs of its ſpherical figure, 108—110. of its annual and diurnal motion, 110—112. 113—118. 239. flattened towards the poles, 235—237.
Eccentricity, 121.
Eclipſe, 139. 145—158.
Ecliptic, 123. 181.
Effect and cauſe, 4. 6.
Elaſticity, 41.
Ellipſis, its deſcription, 97. 122.
Elongation, 105. 113—118.
Equator, 181. meaſure of a degree at, 236.

Equilibrium,

Equilibrium, what, 53.
Equinox, 187. preceſſion of, 240.
Eſcapement in clocks, 88, 89. dead beat, 89. free, 91.
Extenſion, 7, 8, 9 & ſeq.
Eye, 259. See *Viſion*.

F

Figure or form, 9.
Final cauſes, 162. of the planets, 164.
Firſt cauſe is intelligent, 103. 163.
Fixed ſtars, 103. 108. have no parallax, 130. changes obſerved among them, 189—196. are either falling or move in immenſe orbits, 217. change their longitude, 240. their aberration, 256.
Focus of light, 316.
Force, or momentum, 19. accelerating or retarding, 35. 36. diſpute concerning the meaſure of force, 37—39. centripetal, 94 & ſeq.

G

Georgium ſidus, 124.
Gold, its great ductility, 12.
Gravitation, 26 & ſeq. center of, 74—79. its univerſality deduced from its effects, 207—217.
Gravity, See *Gravitation*.

H

Halo, 280.
Halley, Dr. 132.
Hard bodies, the conſequence of their ſtroke, 33. none are found in nature, 34.
Helix, 68, 69.
Herſchel, Mr. 124. 193. 358.
Horizon, 181.

I

Incidence, angle of, 261.
Inclined plane, 64—66.
Inertia, 9. 18.
Inflection, 298 & ſeq.
Intelligence finite, 1. 101.
Intuition, 2.
Impenetrability, 7, 8. 13—& ſeq.
Impulſe, 33. is greater than preſſure, 34.
Irregularities in the planetary ſyſtem, 218 & ſeq.
Jupiter, 118—120. its moons, 139. 324.

K

Knowledge, 1. 3—5. mathematical, why ſo perſpicuous and certain, 4. philoſophical, the nature of its proofs, 5. how acquired, 100.

L

Latitude, 182.
Lenſes, 332.
Lever, 55 & ſeq.
Light, the quantity received by oblique ſurfaces, ſhewn, 185. in what it conſiſts, 254. its velocity, 254. is uniform, 256. its momentum, 257. rarity, 258. various refrangibility and colour, 264 & ſeq. various reflexibility, 270 & ſeq. alternate reflection and tranſmiſſion, 280 & ſeq. its inflection, 298 & ſeq. powers by which its reflection and refraction are effected, 304——314.
Longitude, 182.

M

Magic lanthorn, 341.
Magnitude apparent, 335.
Maintaining power in clocks, 88.
Mars, the planet, 124. teleſcopic appearance, 177.
Matter, and its properties, 7 & ſeq.
Mathematical knowledge, why perſpicuous and certain, 4. ſuppoſes extenſion to be analyſed, 9.
Mechanical powers, enumerated, 54. their general properties, 72—74.

Medium,

Medium, 260. various, diſperſe light unequally, 269.
Mercury, the planet, 107, 176.
Micrometer ſcrew, 71.
Microſcopes, 336——340.
Mirrors, 263. 317. 355. burning, 319.
Moon, eclipſe of, 110. other phenomena, 141—158. teleſcopic appearance, 168—173. cauſe of its motion, 208. its irregularities explained, 218—232. colour during an eclipſe, 297.
Momentum, 19. of light, 257.
Motion, 7, 8. quantity of, 19. abſolute, relative and apparent, or angular, 20. laws of, 21—25. compoſition and reſolution of, 23, 24. uniformly accelerated, 27—31. and retarded, 31, 32. how produced, deſtroyed or changed, 33—44. diſpute concerning the meaſure of its quantity, 37—39.

N

Natural philoſophy, its proofs, 5. how acquired, 100.
Nebulæ, 193.
Newton, 103. 291. 315. 350. 351.
Nodes, 123.
Nutation, 240.

O

Opacity, its cauſe, 294.
Orbits, 197—207.

P

Parallax, annual, 120. 139. general account of, 128. horizontal, 129.
Paul's church, St. its colour at ſunſet, 297.
Pencil of rays, 316.
Pendulums, 26. 80 & ſeq. with a deal rod, 91. gridiron, 91—93. vibrate ſlower near the equator, 236.
Perihelium, 122.
Philoſophizing, rules of, 6.
Philoſophy, natural, its proofs, 5. how acquired, 100.
Planets, primary, 105. their mutual appearances, 113—115. their orbits elliptical, 121. nearly circular, 122. and inclined to the ecliptic, 123. their number and affections, 124—127. moons, 137. motions explained, 207. irregularities, 232.

Planets, secondary, 137. their motion explained, 208.
Porosity of matter, 17.
Preceſſion of the equinoxes, 240.
Preſſure, is leſs than impulſe, 34.
Pringle, Sir John, 103.
Priſm, 264 & ſeq.
Projectiles, 97.
Pulley, 61—64.

R

Radiant point, 316.
Rainbow, 273 & ſeq.
Reflection of light, defined, 261. analogous to refraction, 292. regular, 317.
Reflexibility, 270.
Refraction, 261. regular, 320.
Refrangibility, 264.
Repulſion, 47. apparently exerted between floating bodies, 50.
Retarded motion, 31, 32.
Revolving bodies, 94—99.
Right aſcenſion, 182.
Romer, Monſ. 254.
Rules of philoſophizing, 6.

S

Saturn and its ring, 137. 179.
Screw, 68 & ſeq.
Seaſons, 180—188.
Shadow of the earth, 110.
Solſtice, 187.
Spectacles, 333.
Speculum. See *Mirror*.
Spectrum, 266 & ſeq.
Stars, fixed. See *Fixed Stars*.
Sun, 104. its parallax, 131. teleſcopic appearance, 174. its atmoſphere, 175.
Swing-wheel, 88.
Syntheſis and analyſis, 100.

T

Tackle, 61—64.
Teleſcopes, refracting, 342. their imperfections, 348. achromatic, 351—353. reflecting, 358.
Termination, 183.
Tides explained, 241—251.
Tranſit of Venus over the Sun's diſc, 132—136.
Tranſparency, its cauſe, 294.
Tropics, 188.

U

Undulation of the air, 345,
Univerſe, ſyſtem of, 174.

V

Variation of the moon's motion, 144.
Velocity, final, 29, 31. initial, 32.
Venus, its apparent motions and proportional diſtance from the ſun, 105—107. 113. tranſit over the diſc of the Sun, 132—136. teleſcopic appearance, 176.
Viſion, 258. 327 & ſeq.

W

Wedge, 66—68.
Weight, 35.
Whitehurſt, Mr. his inſtrument for meaſuring the time employed by falling bodies, 42.

Y

Year, *natural*, 187. natural, periodical, and ſideral, 220.

Z

Zenith, 182.